Math 174, Lecture 1

1. Pass out and discuss course syllabus.
   - classroom changes on Tuesday (for computer use)
   - first test may not be before drop day
2. class photos
3. Homework: pages 15-16, numbers 1, 3, 6, 8, 12, 14, 17, 18, 20, 23, 24, 27, 29, 38
   \textbf{Note:} Not collected. Ask questions in class.
4. Quiz: Next Thursday (on above homework).
5. Topics for course (logic, number theory, proof techniques, combinatorics)
7. \textbf{Definition:} A statement is a sentence which is either true or false but not both.
   \textbf{Examples:} (1) Six is one less than five.
   (2) Six is one more than five.
   \textbf{Non-Examples:} (1) $x > 5$
   (2) The boy is in this classroom.
8. \textbf{Definition:} If $p$ is a statement variable, the negation of $p$ is “not $p$” (denoted $\sim p$).
   \textbf{Example:} The negation of “cold” is “not cold” (note this counts lukewarm; also we have substituted a value for the variable).
9. \textbf{Definition:} If $p$ and $q$ are statement variables, the conjunction of $p$ and $q$ is “$p$ and $q$” (denoted $p \land q$).
   \textbf{Example:} The conjunction of $x > 0$ and $x \leq 2$ is $0 < x \leq 2$.
   \textbf{Question:} What does it take for $p \land q$ to be true?
10. \textbf{Definition:} If $p$ and $q$ are statement variables, the disjunction of $p$ and $q$ is “$p$ or $q$” (denoted $p \lor q$).
    \textbf{Examples and Non-Examples:} (1) “Do you want cream or sugar with your coffee?” (inclusive)
    (2) “Do you want coffee, tea, or milk?” (exclusive (?))
    \textbf{Question:} What does it take for $p \lor q$ to be true?
11. \textbf{Definition:} A statement form is an expression made up of statement variables (like $p$ and $q$) and logical connectives (like $\sim$, $\land$, and $\lor$) that becomes a statement when actual statements are substituted for the variables.
    \textbf{Examples and Non-Examples:} (1) $\sim (p \land q)$
    (2) $\sim p \land \sim q$
12. Order of operations: $\sim$ before $\land$ and $\lor$
13. Which of these mean the same thing? $\sim (p \land q)$, $\sim (p \lor q)$, $p \land \sim q$, $p \lor \sim q$
    \textbf{What does that last question mean?}
    \textbf{Definition:} Two statement forms $P$ and $Q$ are logically equivalent if they have identical truth values for each possible substitution of statements for their statement variables (denoted $P \equiv Q$).
14. Truth Tables. Illustrate with $\sim (p \land q)$, $\sim (p \lor q)$, $p \land \sim q$, $p \lor \sim q$ (discuss answer to previous questions), $p \land \sim p$, and $p \lor \sim p$. 
15. De Morgan’s Laws: $\sim (p \land q) \equiv p \lor \sim q$
     $\sim (p \lor q) \equiv p \land \sim q$

16. Definitions: A **tautology** is a statement form that is always true regardless of the truth values of the statement variables. A **contradiction** is a statement form that is always false regardless of the truth values of the statement variables.

**Examples:** See above truth tables.

17. Theorem 1.1.1 on Logical Equivalences. Let $p$, $q$, and $r$ be variables, $t$ a tautology, and $c$ a contradiction. Then (for example)
   - $p \land q \equiv q \land p$
   - $(p \land q) \land r \equiv p \land (q \land r)$
   - $p \land t \equiv p$
   - $p \land \sim p \equiv c$
   - $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
   - $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

18. Problems for Further Discussion: pages 15–16, numbers 7 (use of “but”), 28 (De Morgan’s Laws)