Instructions: Answer as many of the problems below as you can. At the end of the time allotted, turn in a list of your answers. Your answers should be expressed in simplest form. Exact answers are required on all problems.

1. What is the maximum value of the function

   \[ 3 \cos \theta - 2 \sin \theta \]?

2. There are 6 people in a room. Each person randomly chooses a positive integer \( \leq 20 \). What is the probability that some two of the people choose the same number? Express your answer as an exact decimal.

3. The largest known explicit example of a prime number is currently \( 2^{6972593} - 1 \). How many digits does it have?

4. Let \( P \) be a point in an equilateral triangle with each side of length 1. Let \( h_1 \), \( h_2 \), and \( h_3 \) be the distances from \( P \) to the three sides of the triangle. What are all the possible values for \( h_1 + h_2 + h_3 \)?

5. Suppose that \( f(x) \) is a polynomial of degree 5 and with leading coefficient 2001. Suppose further that

   \[ f(1) = 1, \quad f(2) = 3, \quad f(3) = 5, \quad f(4) = 7, \quad \text{and} \quad f(5) = 9. \]

   What is the value of \( f(6) \)?

6. Suppose \( x \) and \( y \) are real numbers such that

   \[ 2x^2 + y^2 - 2xy + 12y + 72 \leq 0. \]

   What is the value of \( x^2y \)?

Solutions are located at the website http://www.math.sc.edu/~filaseta/contests/contests.html
7. A triangle has two sides of length 5 and one side of length 6. A rectangle \( R \) is formed with one edge on the side of length 6 and a vertex (or corner point) on each of the other two sides. What is the maximum possible value for the area of \( R \)?

8. Let

\[
N = 10^{96} - 10^{80} + 10^{64} - 10^{48} + 10^{32} - 10^{16} + 1.
\]

Then

\[
\frac{1}{N} = 0.d_1d_2d_3\ldots d_r
\]

(so that the block of digits \( d_1d_2d_3\ldots d_r \) repeats indefinitely). There is more than one value of \( r \) for which such digits \( d_1, d_2, \ldots, d_r \) exist. If the smallest such \( r \) is \( k \), then it is known (and not so hard to show) that every such value of \( r \) is divisible by \( k \). What is the value of \( k \)?