Team Problems November 15, 2003

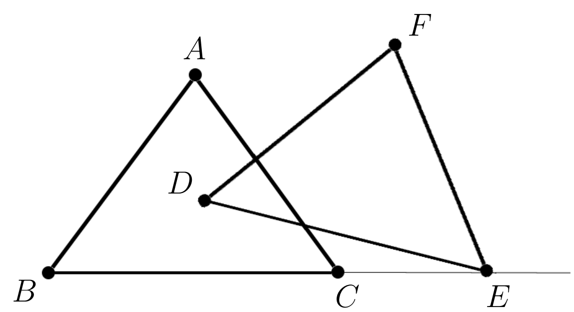
Instructions: Answer as many of the problems below as you can. At the end of the time allotted, turn in a list of your answers. Your answers should be expressed in simplest form.

1. Compute

$$(\log_2 3)(\log_3 4)(\log_4 5)\cdots(\log_{2002} 2003)$$

accurate to 4 digits after the decimal point.

2. In the figure to the right (not drawn to scale), $\triangle ABC$ and $\triangle DEF$ are equilateral triangles each having side lengths equal to one unit. If D is in the center of $\triangle ABC$ and E is on line $\overleftrightarrow{BC}$, then what is the distance from C to E? Give an answer accurate to 6 digits after the decimal point.



3. Two players play a game on the board below as follows. Each person takes turns moving the letter **A** either downward at least one rectangle or to the left at least one rectangle (so each turn consists of moving either downward or to the left but not both). The first person to place the letter **A** on the rectangle marked with the letter **B** wins. How should the first player begin this game if we wants to assure that he wins? Answer with the number given on the rectangle that he should move the letter **A** to.

1	2	3	4	5	6	7	8	**A**
								9
								10
								11
B								12

Solutions are located at the website http://www.math.sc.edu/~filaseta/contests/contests.html

4. Find every positive integer N for which $2003 \le N \le 2500$ and $x^4 - y^4 = N$ holds for some integers x and y.

5. Two positive integers a and b each ≤ 100 are chosen independently at random (so it is possible that $a = b$). What is the probability that $\gcd(a, b)$ (the greatest common divisor of a and b) is divisible by at least one of 11, 13, 23, and 31? Give an exact answer written in decimal notation.

6. The polynomial

$$x^{2003} - x^{2002} - 2003\,x^{2001} - x^2 - 2003$$

has one positive real root. What is the value of this root accurate to 5 digits after the decimal point?

7. How many consecutive zeroes appear after the decimal point and before the first non-zero digit after the decimal point in the decimal expansion of

$$\sqrt{2^{2004} + 1}\ ?$$

8. The equation

$$\frac{1!}{2003!} + \frac{2!}{2004!} + \frac{3!}{2005!} + \frac{4!}{2006!} + \cdots + \frac{2003!}{4005!} + \frac{2004!}{4006!} = A \cdot \left(\frac{1}{2002!} - \frac{2005!}{4006!} \right),$$

holds for some rational number A. Find the value of A and express your answer in the form a/b where a and b are positive integers < 4000.