Institutional Influences on Chinese Mathematics

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Instructor: Professor Moon Duchin
Erik Palmer
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Any inquiry into the nature of mathematics in China before the arrival of western ideas will yield an observation of the distinctly Chinese flavor of numeric and algebraic approaches which permeates throughout all its math. An example which immediately illustrates this difference can be found in the two different approaches to one of the most famous and respected theorems in math, Pythagoras's theorem. In the theorem Pythagoras addresses the issue of finding the length of a the hypotenuse of a right triangle from its two sides through the use of geometric shapes. Pythagoras succeeds in achieving a geometric proof which does not rely on the properties of the numbers involved, but on the properties of the geometric shapes and lengths believed to be separate from numbers. In contrast, the Chinese came to the same $a^2+b^2=c^2$ conclusion, however their method inextricably combines numeric computation in the pursuit of finding the relationship. The numerical properties of this method, called '*gougu*', are best illuminated by a direct quote from an early Chinese text on mathematics and astronomy, the *Zhoubi suanjing*. Contained in what is believed to be its oldest section, there is the explanation of the *gougu* method:

> "Thus, let us cut a rectangle (diagonally) and make the width 3 (units) wide, and the length 4 (units) long. The diagonal between the two corners will then be 5 (units) long. Now after drawing a square on this diagonal, circumscribe it by half-rectangles like that which has bee left outside, so as to form a plate. Thus the (four) outer half-rectangles of width 3, length 4, and diagonal 5, together make two rectangles (of area 24); then (when this area is subtracted from the square plate of area 49) the remainder is of area 25. [see fig.1]" (Needham 23)

Here it can be seen that, while the Chinese do use geometric figures in their method, they are primarily concerned with them as a means of discerning the numerical relationship of the numbers. It is indeed exactly this focus on the numerical algebraic relationships and computation which characterizes the flavor of the mathematics that will permeate through all of China's math for the next 1500 years.

At this point one might be tempted to ask, in a culture that spans 3,000 years, how could the entirety of their mathematical focus be restricted to the narrow domain of math dependent on algebraic and numeric computation? The answer surely involves a throughout examination of many factors of Chinese culture and thought, however a large part of this pervasive trend was a result of extensiveness with which the government relied on and combined with mathematics. In a quote from the commentaries within the *Zhoubi suanjing,* the connection between the *gougu* method and government becomes undeniable:

> "Emperor *Yu* quells floods, he deepens rivers and streams, observes the shape of mountains and valleys, surveys the high and low places, relieves the greatest calamities and saves the people from danger. He leads the floods east into the sea and ensures no flooding or drowning. This is made possible because of the g*ougu* theorem..." (Crossley Lun 28-29)

From the above examples, it can be seen that Chinese math from its beginnings was inherently numerical and algebraic, and that math itself was intertwined with ideals of government and ruling. Therefore it is the aim of this paper to demonstrate that the institutions in place throughout the history of China were responsible for restricting Chinese math to numerical and algebraic modes of thought through the creation of a mathematical cannon and official patronage of mathematicians as astronomers. Proof of this influence can be found in the numerical nature of the all the mathematical achievements we now attribute to Chinese math, such as in the example of Chinese computations of the value of π . However, a quick survey of some historical mathematical works will show that these achievements were the result of the creation of a mathematical cannon responsible for fostering the numerical basis for math in all of China.

Every mathematical tradition demands a primary source, or classic which describes the standard to which everything else is measured. In the case of Greek math, Euclid elements was such a classic in that it outlined the methods and style which Greek math would advance. In the case of China, with its Confucian scholar traditions, the premise of a single work driving all mathematics which would follow it was even more likely. According to Sal Restivo, *The Nine Chapters on Mathematical Art* was just such a classic:

> "...[*The Nine Chapters on Mathematical Art*] had the same sort of influence that Euclid's *Elements* had on Greek mathematics, even to the extent of inhibiting mathematical developments. This was a result of the degree to which it systematized mathematics and became a 'classic'. It was studied by a wide range of scholars. Liu Hui's commentary [on the *Nine Chapters*] in particular influenced mathematics for more than a millennium." (24)

Designed as a handbook for architects, officials, engineers and merchants the *Nine Chapters* contained 246 problems. *A*mong other things the work consists of problems dealing with finding areas, volumes, percentages and proportions, distributions, simultaneous linear equations with positive and negative numbers, and applications of the *gougu* method. It is important to note that evident from the style of the problems contained in the book all the way down to the algebraic methods of computation in which they were solved, there is the flavor of necessary numeric application to real world numbers. That is none of the problems focused on abstract proofs with reasoning, or iterated general theories. They instead were all math problems approached algebraically and had numeric answers.

Scholars debate the exact date of the origin of *The Nine Chapters of Mathematical Art,* placing it somewhere between 300 B.C.E. - 100 C.E.. However there is no debate on the extensiveness of its influence on the coming centuries. After its conception, *The Nine Chapters* was institutionalized as a classic and therefore became required reading for mathematicians in China. Furthermore, evidence that its influence did not decline with age can be found in a quote from Restivo, "According to official regulations [in the *Tang* dynasty (618-907 C.E)], mathematics students had to spend three years studying Liu Hui's (3rd c. C.E.) works [including his annotation of *The Nine Chapters*]. His works were also prescribed texts in the Japanese schools opened in 702." (25) The fact that the government accepted and made the *Nine Chapters* central to its mathematical requirements, shows that it became part of the mathematical institutions of China. The result that then followed as a consequence, was that every new scholar studying math in China would learn the algebraic and numerical methods in the *Nine Chapters* as the sole acceptable basis for mathematics.

Prescriptions such as the aforementioned, no doubt became more common as a result of government patronage creating a need for trained mathematicians. In most cases, mathematicians were in fact solicited by the state to do astronomical predictions and reform the official calender as Needham states:

> "It would be hard to find a mathematician in the *Chhou Ren Chuan* [*Chou Ren Zhuan*] (Biographies of Mathematicians and Astronomers, 1799 C.E.) who was not called upon to remodel the calender of his time, or to help in such work. For reasons connected with the ancient corpus of cosmological beliefs, the establishment of the calender was the jealously guarded prerogative of the emperor, and its acceptance on the part of tributary states signified loyalty to him. When rebellions or famines occurred, it was often concluded that something was wrong with the calendar, and the mathematicians were asked to reconstruct it." (152 v.3)

As the development of math is influenced by social factors, the cohesion of mathematicians and calender reform pushed mathematical achievements in a numerical direction. The result was math that excelled in practical applications and numerical approximations, but lacked abstractions and an emphasis on a systematic form of logic. An example of the type of achievements that mathematicians working in service of the government as astronomers produced can be seen in Zu Chongzhi (430-501 C.E.) approximations to π. In his work *Zhui Shu* gives upper and lower bounds (3.1415927 and 3.1415926) with an accuracy unmatched until Vieta in 1593 C.E. Unfortunately his *Zhui Shu* which contained the methods of his calculation have since been lost. However, from the numerical nature of the subject matter, it is possible discern that Zu no doubt used an algebraic and numerical approach to approximate π. With little effort numerous other works demonstrating the talent and ingenuity of Chinese mathematicians can be found, but just like Zu approximations, they are all confined to the realm of algebraic manipulations and computations involved in solving numerical problems.

In the following centuries government patronage of astro-calendric calculations continued to codify the algebraic and numerical nature of Chinese mathematicians and their achievements. A particular event highlighting style of thought of Chinese mathematicians derived from this tradition is a predominate figure's response to the coming of the Jesuits and their suggestions for calender reform. In the 17th century C.E., Mei Wending was a mathematician who gained fame for combining the western math being imported into China which ancient Chinese methods, thus simultaneous reconciling the two and becoming know as the figure responsible for reinvigorating Chinese mathematics. He believed that the western methods the Jesuits brought with them were derived from ancient Chinese methods, and that it was therefore necessary to reunite the numerical emphasis of Chinese methods with the Jesuit's math. About the Jesuits methods he said this:

> "They never took time to inquire deeply into the origins and vicissitudes of Chinese calculation[methods]. They precipitously passed on the shallow techniques of their day and referred to them as the culmination of the ancient *Nine Chapters*[classic]. Consequently, they slighted ancient methods as not worth taking seriously." (Elman 155)

From his criticisms one can see the influence of the algebraic numeric tradition on his point of view by his insistence for the need of numerical methods to legitimize the more accurate theories of the Jesuits.

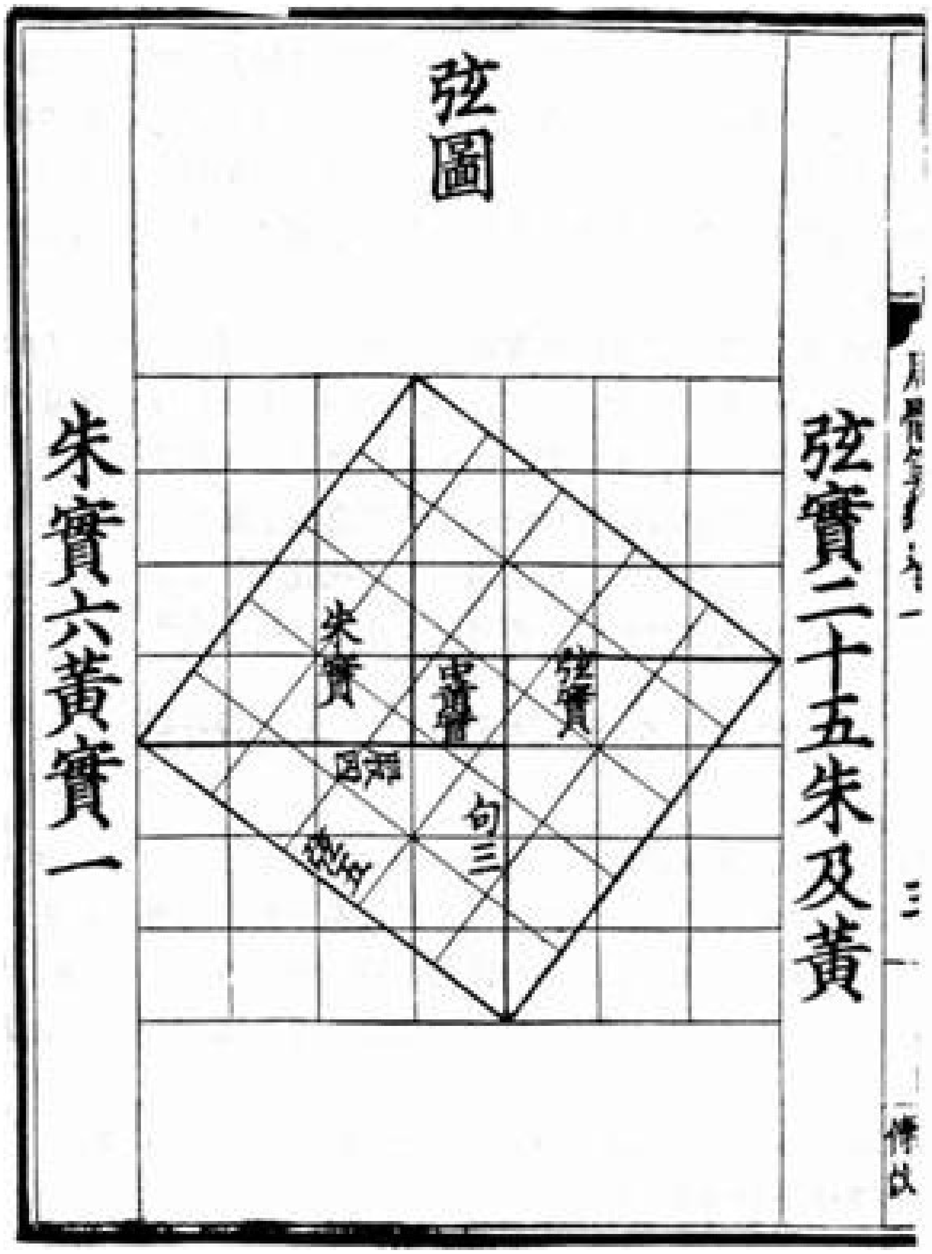
Moreover, scholars have pointed directly to this need for calendar reform as being a factor in limiting Chinese math to its numeric and algebraic characteristics. Needham asserts that, "It has been thought that this preoccupation fixed them irretrievably to concrete number, and prevented the consideration of abstract ideas..." (152) Here again, if we see Mei Wending as a figure standing at the end of the culmination of a long Chinese tradition of numeric and algebraic method, we find further evidence of the conviction of Chinese mathematicians that there necessarily exists a connection between numeric computation and astronomy. In his statement, Mei explains:

> "...Mathematical astronomy requires numbers. Outside of the numbers there are no principles, and outside of the principles there are no numbers. Numbers are the orderly demarcation of principles. Numbers cannot be spoken of arbitrarily, but principles at times can be talked about via vague images. Hence, arbitrary views have been associated [with principles], which have deluded the people and brought chaos to heaven's regularity. All this results from not obtaining the true principles and numbers instead maliciously overturning reality." (Elman 159)

Although Mei speaks here in response to the question of astronomy, it is clear from his answer that his statement has far reaching implications to all of Chinese math. In spelling out that all mathematical principles necessitate a numeric and algebraic relationship, he consequently denounces areas of math that would abstract themselves from numeric computation. In relation to Chinese mathematics as a whole, his views essentially sum up the nature of Chinese math up to that point in history.

In conclusion, we can see that while there no doubt exists a number of cultural and psychological issues influencing the development of Chinese math, an overwhelming factor was government institutional influence. In China, the creation of classic works such as the *Nine Chapters* and their institutional backing combined to make a mathematical cannon, through which all preceding math would be filtered and measured by. Furthermore, state patronage of astronomers kept most mathematicians employed in astro-calendric approximations continuing to keep them focused on and based in the algebraic and numerical methods. Finally, these characteristics became so intertwined with the concept of math for the Chinese that they could not accept any abstraction that lacked a direct relationship with numerical methods. For these reasons it is evident that due to the government institutionalization of math and mathematical classics from the very beginning of history, Chinese math was irrevocable tied to algebraic and numerical calculations.

Fig.1 *Gougu*



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