Chapter 12
Section 12.5

Equations of Lines

1. The **vector equation** of a line is the most basic and easiest to remember. Given a point on a line r_0 and a directional vector v every point on the line as a representation as

$$r = r_0 + t \quad \text{for some } t \in \mathbf{R}.$$

2. The **parametric equations** of a line follow immediately from this. They are just the invidual equations for the x, y and z coordinates.

3. The **symmetric equations** of a line follow from the parametric equations and are obtained by setting all the parametric equations equal to the parameter t.

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.$$

Note: We can restrict the parameter t to create line segments instead of lines.

Equations of Planes

Unfortunately, one vector in a plane is not enough information to completely to define a plane like it is for a line. However, if we find a vector orthogonal to a plane, as well as a point in the plane, it is enough information to completely define a plane. This orthogonal vector is called the **normal vector** and is usually denoted with n.

1. If r, r_0 are vectors in the plane then $n \cdot (r - r_0) = 0$ or $n \cdot r = n \cdot r_0$. This is called the **vector equation of the plane**.

2. By expanding this dot product we get

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

   which is known as the **scalar equation of the plane.**

3. Noticing that the above scalar equation is a linear equation, the **linear equation of a plane** is

$$ax + by + cz + d = 0.$$

Examples and Applications

1. Find an equation of the plane through the point $(2, 4, -1)$ with normal vector $n =< 2, 3, 4 >$ and find the intercepts.

2. Find an equation of the plane that passes through the points $P(1, 3, 2), Q(3, -1, 6)$ and $R(5, 2, 0)$.

3. Find the point at which the line with parametric equations $x = 2 + 3t, y = 4t, z = 5 + t$ intersects the plane $4x + 5y - 2z = 18$.

4. Find that angle between the planes $x+y+z = 1$ and $x-2y+3z = 1$. Then find the symmetric equations for the line of intersection.

5. Find a formula for the distance D from a point $P(x, y, z)$ to the plane $ax + by + cz + d = 0$.

6. Find the distance between the parallel lines $10x + 2y - 2z = 5$ and $5x + y - z = 1$.