Speaker: Fred Stoll (USC)

Title: Littlewood–Paley Theory for Subharmonic Functions on the Unit Ball in $\mathbb{R}^N$

Abstract: The seminar talk will be a continuation of the lectures given in the fall concerning the Lusin area integral and Hardy type spaces of subharmonic of subharmonic functions. The talk will be self contained. Let B denote the unit ball in $\mathbb{R}^N$ with boundary S. A C^2 function f is subharmonic on B if $\Delta f \geq 0$ where Δ is the usual Laplacian on $\mathbb{R}^N$. We introduce the function g_λ^*, $\lambda > 1$, defined for a nonnegative C^2 subharmonic function f by

$$g_\lambda^*(\zeta, f) = \left[\int_B (1 - |y|) \Delta f^2(y) K_\lambda(y, \zeta) dy \right]^{\frac{1}{2}},$$

where

$$K_\lambda(y, \zeta) = \frac{(1 - |y|)^{(\lambda-1)(N-1)}}{|y - \zeta|^{\lambda(N-1)}}.$$

For $\lambda = N/(N-1)$ one obtains the function g^* of Littlewood and Paley. In the talk we will prove that the inequality

$$\int_S [g_\lambda^*(\zeta, f)]^p \, d\sigma(\zeta) \leq C_p \sup_{0<r<1} \int_S f^p(r\zeta) d\sigma(\zeta)$$

holds for all $\lambda \geq N/(N-1)$ when $p \geq 2$, and for $\lambda > 3 - p$ whenever $1 < p < 2$. Taking $\lambda = N/(N-1)$ proves that $\|g^*(\cdot, f)\|_p \leq C_p \|f\|_p$ for all $p > (2N-3)/(N-1)$.