Speaker: Fred Stoll (USC)

Title: Littlewood-Paley Theory for Subharmonic Functions on the Unit Ball in $\mathbb{R}^N$ (Part 2)

Abstract: Let B denote the unit ball in $\mathbb{R}^N$ with boundary S. For a non–negative C^2 subharmonic function f on B and $\zeta \in S$, the Lusin square function $S_\alpha(f,\zeta)$ is defined by

$$S_\alpha(f,\zeta) = \left[\int_{\Gamma_\alpha(\zeta)} (1-|x|)^{2-N}\Delta f^2(x)dx\right]^{\frac{1}{2}},$$

where for $\alpha > 1$, $\Gamma_\alpha(\zeta) = \{x \in B : |x-\zeta| < \alpha(1-|x|)\}$ is the non–tangential approach region at $\zeta \in S$, and Δ is the Laplacian in $\mathbb{R}^N$. In the seminar we will prove the following: **Theorem.** *Let f be a non–negative subharmonic function such that f^{p_o} is subharmonic for some $p_o > 0$. If*

$$\|f\|_p^p = \sup_{0<r<1}\int_S f^p(r\zeta)d\sigma(\zeta) < \infty$$

for some $p > p_o$, then for every $\alpha > 1$,

$$\int_S S_\alpha^p(f,\zeta)d\sigma(\zeta) \le A_\alpha \|f\|_p^p$$

for some constant A_α independent of f.

The above result includes the known results for harmonic or holomorphic functions in the Hardy H^p spaces, as well as for a system $F = (u_1,\ldots,u_N)$ of conjugate harmonic functions for which it is known that $|F|^p = \left(\sum u_j^2\right)^{p/2}$ is subharmonic for $p \ge (N-2)/(N-1), N \ge 3$. We will also consider the reverse inequality

$$\|f\|_p^p \le A_\alpha \int_S S_\alpha^p(f,\zeta)d\sigma(\zeta)$$

when $p > 1$.