MATH 141, Practice Exam III
Prof. Joshua Cooper, Fall 2009

Name: ____________________________

Each problem is worth 10 points, for a total of 100. Show all your work, fully simplify your answers, be careful with the details, and good luck!

1. Find the antiderivative $F(x)$ of $f(x) = x^2 - x + 1 - x^{-1}$ with $F(1) = 1$.

2. Estimate the area under the curve $f(x) = \sin(x)$ from $x = 0$ to $x = \pi$ by taking a Riemann sum with $n = 6$ and using the left-endpoint rule.

3. Express the following as a definite integral:
   \[ \lim_{n \to \infty} \sum_{i=1}^{n} \frac{\sin \left( \pi + \frac{\pi i}{n} \right)}{n + i} \]

4. Place the following quantities in order from smallest to largest.
   (a) $\int_{0}^{\pi} \sin x^2 \, dx$
   (b) $\int_{-3}^{3} (\sin x)e^{-x^2} \, dx$
   (c) $\int_{0}^{5} (2 - 2x) \, dx$
   (d) $\int_{0}^{\pi} \sin \frac{x^2}{1 + x^2} \, dx$

5. Find the derivative of the function $F(x) = \int_{x^2}^{3} \sqrt{1 + t^2} \, dt$.

6. Find the value of $\int_{0}^{2} (y + 1)(2y^2 - 1) \, dy$.

7. The velocity of a particle at time $t$ is given by $\frac{1}{1 + t^2} - \frac{3}{4}$. Find the net displacement and total distance traveled over the time interval $[0, 1]$.

8. Find the value of
   \[ \int_{1}^{2} \frac{e^{1/x} - x^2}{x^2 e^{1/x} + x^3} \, dx \]

9. Find the area of the region enclosed by the curves $x^4$ and $2x^2 + 1$.

10. Evaluate the indefinite integral $\int \frac{\sqrt[3]{1 + \tan(x)}}{\cos^2 x} \, dx$. 