Integration Methods II: Partial Fractions and Trig Substitutions

Douglas Meade, Ronda Sanders, and Xian Wu

Department of Mathematics

Overview

The objective of this lab is to use Maple to practice integration skills using partial fraction decomposition and trigonometric substitutions.

Maple Essentials

- Important Maple commands introduced in this lab:

Command/Example	Description

`convert(f(x), parfrac, x);`
 Example:
 `convert(x/((x^2+1)*(x-2)^3)),parfrac,x);`
	Converts a rational function $f(x)$ into its partial fraction form.

`completesquare(f(x),x);`
 Example:
 `completesquare(x/(x^2+2*x+2),x);`
 Note: *You need to load the* `student` *package before using this command.*
	Completes the square to rewrite a quadratic expression.

Note: *You may omit the last* `x` *in the above commands if the independent variable is obvious.*

- The *Trig Substitution* maplet is available on the course website:

  http://www.math.sc.edu/calclab/142L-S11/labs → Trig Substitution

  This maplet is designed to help you practice the steps involved in integration using trigonometric substitutions. Click on **New Integral** to have the maplet generate a problem for you to practice. Next, identify the type of trigonometric substitution and follow the steps to solve the problem. (You will need a pencil and paper to do all the steps yourself.) You can click **Hint** or **Show** if you need help or want to skip a step.

- The *Partial Fractions: General Decomposition* maplet is available on the course website:

  http://www.math.sc.edu/calclab/142L-S11/labs → Partial Fractions: General Decomposition

  This maplet is designed to help you practice partial fraction decomposition. Click on **New Function** to have the maplet generate a problem for you to practice. Then add terms one at a time until you have the general form for the decomposition. Once you are confident, click on the **Include Completing the Square** box to make the decomposition more difficult.

- The *Partial Fractions: Evaluating the Integral* maplet is available on the course website:

  http://www.math.sc.edu/calclab/142L-S11/labs → Partial Fractions: Evaluating the Integral

  This maplet is designed to help you practice evaluating the integral after using partial fraction decomposition. Click on **Modify or Make Your Own Problem** to enter an integral. Click on **Show Partial Fraction Expansion**. Then evaluate the expanded integral and check your work.

Related course material

   Stewart text: §7.3 and §7.4. Calculus text: §6.5.

Activities

1. Launch the *Partial Fractions: General Decomposition* maplet from the course web page and do a few practice problems.

2. Follow the steps below to integrate the following rational functions:

   (a) $f1(x) = \dfrac{1}{(x+a)(x+b)}$. (Page 482, ex. 14)

   (b) $f2(x) = \dfrac{x^3 - 2x^2 - 4}{x^3 - 2x^2}$. (Page 482, ex. 15)

   (c) $f3(x) = \dfrac{x^3 + 4}{x^2 + 4}$. (Page 482, ex. 21)

   (d) $f4(x) = \dfrac{x^2 - 2x - 1}{(x-1)^2(x^2+1)}$. (Page 482, ex. 28)

   (e) $f5(x) = \dfrac{x + 4}{x^2 + 2x + 5}$. (Page 482, ex. 29)

   (f) $f6(x) = \dfrac{x^3 + 2x^2 + 3x - 2}{(x^2 + 2x + 2)^2}$. (Page 482, ex. 38)

Steps:

   The following steps would be used to integrate $f1(x)$.

   (a) Load the maple `student` package.
      $>$ `with(student):`

   (b) Assign the function using the arrow notation.
      $>$ `f1 := x -> 1/((x+a)*(x+b));`

   (c) Write down the general form for the partial fraction decomposition of the function by hand. Then use Maple to complete the expansion and compare it to your general form.
      $>$ `convert(f1(x), parfrac, x);`

   (d) Use a pencil and paper to integrate the function. Then use Maple to complete the integration and check your answer.
      $>$ `int(f1(x),x);`

   (e) You can also use the *Partial Fractions: Evaluating the Integral* maplet to check your integration.

3. Launch the *Trig Substitution* maplet from the course web page and do a few practice problems. Remember, you will need a pencil and paper to complete all the steps by yourself.

Assignment

   Use Maple to complete exercise 15 on page 108 of CalcLabs with Maple. Also, complete exercises 34 and 67 on pages 482-483 of the Stewart textbook. You do NOT have to evaluate the integrals by hand, but show the partial fraction decomposition for all exercises before integrating.