

# Integration Methods: by Substitution and by Parts

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## Overview

The objective of this lab is to provide some Maple tools to help you practice integration skills, such as integration by substitution and integration by parts.

## Maple Essentials

- Important Maple commands introduced in this lab:

Command/Example	Description
<code>Hint(Expression);</code> <b>Examples:</b> <code>r1:=Hint(Int(ln(x),x));</code> <code>P=Int(x*exp(x),x=0..1); Hint(P);</code> <b>Note:</b> You need to load the <code>Student[Calculus1]</code> package before using this command.	It returns <code>Hints</code> for a calculus problem matching the <code>Expression</code> which is typically in form of <code>Diff(...)</code> , <code>Int(...)</code> , or <code>Limit(...)</code> . Note that the first letter of <code>Hint</code> , <code>Diff</code> , <code>Int</code> , and <code>Limit</code> must be capitalized.
<code>Rule[rule](Expression);</code> <b>Examples:</b> <code>Rule[parts,ln(x),x](Int(ln(x),x));</code> <code>Rule[r1](P);</code> <code>Rule[r1[2]](P);</code>	It is in general used to apply a rule obtained from <code>Hint</code> to a matching problem. Therefore, it is often easier to use equation labels or assigned names as inputs.

Maple is very capable in evaluating integrals, derivatives, and limits. However, if you want to see individual steps or methods to be applied for such calculus problems (rather than final answers), then you need to use commands `Hint` and `Rule`. They are very useful when you need a quick hint or want to check your work (not just the answer) for a difficult problem. However, there are a few things that you should know:

1. There are many different ways to do a calculus problem and Maple's way may be different from yours. While Maple in general will give good hints, you may find a different or better way to do the problem.
  2. Because of the constant  $c$  in integral formulas, your final answer to an indefinite integral may look different from the one given by Maple. The best way to be sure that the both are correct is to take the derivative of the difference to see if it equals to zero.
  3. Maple may show some methods that we have not learned yet. Both commands can also be applied to other problems such as finding derivatives and limits.
  4. Since you do need to know how to calculate integrals by hand when taking tests, don't just let Maple do problems for you. Instead, it should be used as a tool to help you to understanding what is taught in class.
- The *Integration by Substitution* maplet is available from the course website:  
<http://www.math.sc.edu/calclab/142L-S09/labs> → Integration by Substitution

This maplet is designed to help you to understand integration by the method of substitutions following individual steps. When clicking **Indefinite Integral** or **Definite Integral**, the maplet will generate a problem for you to practice. Make sure that you type in your answers with correct syntax. For a definite integral, you will be asked to calculate the associated indefinite integral first. You can then select either to change integral limits from the original variable to the new variable or to substitute the original variable back. You can always click **Hint** or **Show** if you need help or want to skip a step.

- The *Integration by Parts* maplet is available from the course website:  
<http://www.math.sc.edu/calclab/142L-S09/labs> → Integration by Parts

This maplet is designed to help you to understand the method of integration by parts following individual steps and its interface is very similar to the last maplet.

### Related course material

§6.3 and §8.2 of the textbook.

### Activities

1. Use Maple to help you to evaluate  $\int x^3 e^{x^2} dx$  (Exercise 27 on page 520):
  - (a) Start a Maple session and don't forget to load `Student[Calculus1]` package.  
`> with(Student[Calculus1]);`
  - (b) Assign the integral to a name, say, `P`. Be sure to use `Int`, not `int` (Can you see why?).  
`> P:=Int(x^3*exp(x^2),x);`
  - (c) Get hint and name it to, say, `hint1`.  
`> hint1:=Hint(P);`  
 The return suggests to use parts with  $u = x^2$  and  $v = e^{x^2}/2$  or a substitution with  $u = x^2$ . Notice that  $u = x^3$  won't work for parts since  $v = \int e^{x^2} dx$  cannot be evaluated.
  - (d) To apply the first hint, enter the following:  
`> Rule[hint1](P);`
  - (e) To apply the second hint, enter the following:  
`> Rule[hint1[2]](P);`
  - (f) You should be able to complete the problem by hand after seeing the hint applied, but you may continue to receive and apply hints if you want to.
  - (g) Your TA may give you more problems to work on. Remember that the commands can also be used for other calculus problems such as finding derivatives and limits. They can also be very helpful later when you need hints for more difficult integrations that will come up in other sections of Chapter 8.
2. Launch *Integration by Substitution* and *Integration by Parts* maplets maplet from the course web and do a few practice problems.

### Assignment

Complete lab activities and review Lab 1 to Lab 5 for Quiz 1 next week.