Series: Convergence Tests

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Overview

The intent of this lab is to introduce a maplet to provide additional practice determining the convergence or divergence of series.

Maple Essentials

- A link to the *SeriesConvergenceTestDrill* maplet can be found on the course website (last column in Lab 12):

  http://www.math.sc.edu/calclab/142L-S09/labs/

The first hurdle in determining the convergence or divergence of a series is to select an applicable test. Then there are steps in each test to be carried out and some of them could easily be overlooked. The best way (and the only way) to overcome those difficulties is to have a lot of practice and this maplet can be very helpful. In Step A, you can input your own series or the maplet will randomly generate one for you to practice. To obtain numerical evidence, you can then choose a range of indices and plot terms and/or partial sum in Step B. Of course, you may go directly to Step C to select an applicable test in best of your knowledge. (You can always try another one if your first choice does not work.) The maplet then opens up a new window and shows you all steps that need to be carried out. If you need a reminder of the test that you are using, click `Help`. This is also a great tool to check your work and answers for homework problems. However, don't depend on it too much as you have to do problems on your own eventually.

Preparation

§10.4, §10.5, and §10.6. Be sure to review steps and to understand conditions needed so that a particular test can be applied.

Assignment

There is no assignment this week so you will have more time to review Lab 7 to Lab 11 for Quiz 2 next week (and complete Project 2 report if needed).

Activities

For each of the following series, decide first which test should be used in determining whether the series diverges or converges and then use *SeriesConvergenceTestDrill* maplet to carry out detailed steps. Try another test if it is not applicable or the answer is inconclusive.

$$(1) \quad \sum_{k=1}^{\infty} \frac{1}{\sqrt{k+1}} \qquad\qquad (2) \quad \sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}} \qquad\qquad (3) \quad \sum_{k=1}^{\infty} \frac{(-1)^k}{\ln(k+1)}$$

$$(4) \quad \sum_{k=1}^{\infty} \frac{(-1)^k}{k\sqrt{k^2+1}} \qquad\qquad (5) \quad \sum_{k=1}^{\infty} \frac{k+1}{k!} \qquad\qquad (6) \quad \sum_{k=1}^{\infty} \frac{(-3)^k}{k!}$$

$$(7) \quad \sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2} \qquad\qquad (8) \quad \sum_{k=1}^{\infty} \frac{(-1)^k(k^2+1)}{2k^2+k-1} \qquad\qquad (9) \quad \sum_{k=1}^{\infty} \frac{2^k 3^k}{k^k}$$

$$(10) \quad \sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+1)(k+2)}} \qquad\qquad (11) \quad \sum_{k=1}^{\infty} \frac{1}{(3k-2)^{k+0.5}} \qquad\qquad (12) \quad \sum_{k=1}^{\infty} \frac{\tan^{-1} k}{k^2+1}$$

$$(13) \quad \sum_{k=1}^{\infty} (-1)^k \left(\frac{k}{k+1}\right)^k \qquad\qquad (14) \quad \sum_{k=1}^{\infty} \frac{(-1)^k 3^k k!}{(2k)!} \qquad\qquad (15) \quad \sum_{k=1}^{\infty} \frac{\ln(k!)}{k^3}$$