Taylor Polynomials II

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Overview

This is a continuation of last week's lab. The Activities and Project explore additional applications and manipulations of Taylor polynomials and their remainders. These questions will be helpful as you begin to study power series.

Maple Essentials

There are no new Maple commands to learn this week.

Preparation

Review last week's lab (Lab J) and the material about Taylor polynomials and remainders.

Assignment

1. Project 2 is due at the beginning of next week's lab. Remember to follow the Project Report Guidelines that were handed out with Lab F (and available on the lab homepage).
2. For Mastery Quiz 9 you will need to answer some questions about Taylor polynomials.

Activities

1. Suppose the 4^{th} Maclaurin polynomial for an unknown function $f(x)$ is

$$p_4(x) = 1 - x + \frac{2}{9}x^2 - \frac{3}{25}x^3 + \frac{4}{49}x^4.$$

   Find $f(0)$, $f'(0)$, $f''(0)$, $f^{(3)}(0)$, and $f^{(4)}(0)$.

   Hint: *Review the definition of the nth Maclaurin polynomial.*

2. (a) Use the 1^{st} Maclaurin polynomial for $f(x) = \sin(x^2) + \cos(x^2)$ to approximate $\int_{-1}^{1} \sin(x^2) + \cos(x^2)\, dx$. That is, compute $\int_{-1}^{1} p_1(x)\, dx$.

   (b) Use the Remainder Estimation Theorem to find a bound for $R_1(x)$. This result will be of the form: $|R_1(x)| \leq \frac{M}{2}|x|^2$ for some value of M.

   (c) Use the result found in (b) to approximate the difference $\int_{-1}^{1} f(x)\, dx - \int_{-1}^{1} p_1(x)\, dx$.

   (d) Repeat (a), (b), and (c) for the n^{th} Maclaurin polynomials for $n = 2$, 4, 8, and 16.

   Note: *The case $n = 4$ is completed on the reverse of this lab as an example.*

   (e) Compare the number of decimal digits to the right of the decimal point are known to be correct based on the Remainder Estimation Theorem with the actual number of correct digits. **Note:** *Use Maple's value for the integral as the exact value.*

   (f) Find the lowest degree Maclaurin polynomial that can be used to approximate $\int_{-1}^{1} f(x)\, dx$ to six (6) decimal places to the right of the decimal point.

3. Repeat 2. for the integral $\int_{-\pi}^{\pi} e^{-x^2}\, dx$.

Example: Activity 2: Case $n = 4$

The following Maple commands follow the instructions of Activity 2, parts (a)-(c) for the 4^{th} Maclaurin polynomial for $f(x) = \sin(x^2) + \cos(x^2)$. Make sure you understand each step.

```
> with(Student[Calculus1]):
> f := sin(x^2)+cos(x^2);
> n := 4;
> pn := TaylorApproximation( f, x=0, order=n );
> i4 := int( pn, x=-1..1 );
> evalf( i4 );
> df := diff( f, x$(n+1) );
> plot( abs(df), x=-1..1 );
> M := eval( abs(df), x=-1.0 );
> R4bound := M/(n+1)! * abs(x)^(n+1);
> int( R4bound, x=-1..1 );
```