

Graphical Analysis in Polar Coordinates

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Overview

One of the most challenging aspects to polar coordinates is being able to visualize the graph of a polar function, $r = f(\theta)$. An animation showing exactly how the curve is traced out as the angle moves through its domain is even more useful than a static graph of the function.

The simplest polar plots can be created with the `plot` command — with one additional argument. To create an animation in polar coordinates it is easier to work with a *parametric form* of the equation. (Parametric curves will be discussed in more detail in Calculus III.)

Related Course Material/Preparation

- §11.1.
- Know the basic conversions between rectangular and polar coordinates:

$$\begin{aligned} r &= \sqrt{x^2 + y^2} & x &= r \cos(\theta) \\ \tan \theta &= \frac{y}{x} & y &= r \sin(\theta) \end{aligned}$$

- Remember that all angles need to be specified in radians.
- Be prepared to create some surprising plots that would be almost impossible to create in rectangular coordinates.

Maple Essentials

- The *PolarCurveID* and *Basic14Polar* maplets are available from the course website (last column in Lab 13):

<http://www.math.sc.edu/calclab/142L-S06/labs>

- New Maple commands introduced in this lab include:

Command	Description
<code>arctan(y, x)</code>	two-argument version of the inverse tangent this is essentially equivalent to <code>arctan(y/x)</code> except that the signs of <code>x</code> and <code>y</code> are used to extend the range from $(-\frac{\pi}{2}, \frac{\pi}{2})$ to $(-\pi, \pi)$; this modification makes the two-argument <code>arctan</code> ideal for converting from rectangular to polar coordinates
<code>plot(..., coords=polar);</code>	plot a function in polar coordinates the most common usage is: > <code>R := 2*cos(4*t)</code> > <code>plot(R, theta=0..2*Pi, coords=polar);</code>
<code>animatecurve</code>	animated sketch of a curve e.g., the limaçon $r = 1 + 3 \sin(\theta)$ could be animated as follows: > <code>R := 1 + 3*sin(t);</code> > <code>animatecurve([R, t, t=0..2*Pi], coords=polar);</code> Note: Execute with <code>(plots)</code> : before using <code>animatecurve</code> .
<code>unassign</code>	remove assignments from a Maple name to prevent the name from evaluating to its value, it is necessary to enclose each name in single quotes, e.g., > <code>unassign('x', 'y', 'r');</code>

Activities

- Convert the following points to polar coordinates: $(2,0)$, $(3,3)$, $(0,2)$, $(-2,3)$, $(-2,-5)$, $(0,-3)$, $(1,-\sqrt{3})$. **Note:** Compare the angles obtained with $\arctan(y/x)$ and $\arctan(y,x)$.
- Create plots of the unit circle, $x^2 + y^2 = 1$, in both rectangular and polar coordinates.
Note: In which coordinate system is it easier to plot the unit circle?

- For each of the curves below:

- Find a parameter interval that traces the curve exactly once.
- Plot the curve in polar coordinates.
- Animate the sketching of the curve.

Hint: A polar function $r = f(\theta)$ can be written in parametric form as $r = f(t)$, $\theta = t$.

Note: Optional arguments to the `animatecurve` command include:

- `frames=num` creates an animation with *num* frames; the default number of frames is 16.
- `numpoints=num` instructs Maple to use *num* points in each frame of an animation; the default number of points is 50.

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| (i) $r = 2 + \sin(\theta)$ | (ii) $r = \cos(4\theta)$ | (iii) $r = 3(1 - \cos(\theta))$ |
| (iv) $r = \sin\left(\frac{\theta}{5}\right)$ | (v) $r = \sin(\theta) + \cos\left(\frac{\theta}{3}\right)$ | (vi) $r = 2 + \sin\left(\frac{5\theta}{3}\right)$ |
| (vii) $r = \ln(\theta)$ | (viii) $r = \frac{\theta}{2}$ | (ix) $r = 1 + (\cos(\theta))^3$ |
| (x) $r = (\cos(\theta))^2$ | (xi) $r^2 = \cos(2\theta)$ | |

- The polar function $r = e^{\cos(\theta)} - 2\cos(4\theta) + \left(\sin\left(\frac{\theta}{4}\right)\right)^3$ is called the “butterfly curve”.
 - Find a parameter interval that traces this curve exactly once.
 - Plot or animate the curve.

Assignment

- There is no assignment this week but you need to complete an end-of-course survey. Your TA has instructions for turning in the survey.
- You have just completed the last math-142 lab. Congratulations and have a great break!