

Applications of the Definite Integral

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Overview

Many problems that involve definite integrals — the volume of a solid with a known cross sectional area, the volume by washers or shells, or the arclength of a curve — all depend on the ability to identify the proper integrand and limits. This lab provides practice with the setting up of the appropriate definite integrals, then uses Maple to actually evaluate the integrals.

Maple Essentials

- Links to the *Average Value of a Function* and *Volumes of Revolution* maplets can be found on the <http://calclab.math.tamu.edu/maple/maplets/> webpage. The direct URLs are
`http://calclab.math.tamu.edu/maple/maplets/AverageValue.maplet`
`http://calclab.math.tamu.edu/maple/maplets/VolOfRev.maplet`
- New Maple commands introduced in this lab include:

Command	Description
<code>value</code>	forces the evaluation of an unevaluated integral, derivative, or limit
<code>VolumeOfRevolution</code>	<p>visualize a solid of revolution and find the definite integral corresponding to either the Methods of Washers or Shells.</p> <p>The general syntax for <i>visualizing</i> a solid of revolution: <code>VolumeOfRevolution(top, bot, x=a..b, output=plot);</code></p> <p>The general syntax for the <i>definite integral for the volume</i> of a solid of revolution: <code>VolumeOfRevolution(top, bot, x=a..b, output=integral);</code></p> <p>The default axis of revolution is the x-axis ($y = 0$), and the volume is obtained by the Method of Washers (Disks). If the axis of revolution is the y-axis ($x = 0$), include <code>axis=vertical</code> as an addition argument in the above command.</p> <p>When the bottom edge of the region is $y = 0$, the second argument (<i>bot</i>) can be omitted, but this is not advised.</p> <p>Note: The <code>VolumeOfRevolution</code> command is in the <code>Student[Calculus1]</code> package.</p>

Preparation

Review average value of a function. Review the slicing process for finding the volume of a solid of revolution by the methods of washers and shells, and of a solid with a known cross-section. (These topics are covered in §§7.2-7.3 and 7.6 in Anton, Bivens, Davis, 8th edition.)

Assignment

1. *Before you begin this lab*, please complete the Pre-Integration Applications Survey at the URL
http://distance-ed.math.tamu.edu/maple/dm152_integration/maple_quiz.htm
 (Use your SAM login name as the login name for the quiz.)
2. For Mastery Quiz 4 you will be asked to find the definite integral for the volume of a given solid and the exact and numeric values for the volume.
3. *After you have completed the Mastery Quiz*, take a few minutes to complete the Post-Integration Applications Survey at the URL
http://distance-ed.math.tamu.edu/maple/dm152_integrationb/maple_quiz.htm

Each student who participates in both surveys will receive 2 bonus lab points.

Activities

1. Use the **Average Value of a Function** maplet to estimate (graphically) and compute (exactly) the average value for each of the following functions on the given interval.
 (a) $f(x) = \frac{1}{x}$ on $[1, 3]$ (b) $f(x) = \frac{x}{(5x^2 + 1)^2}$ on $[0, 2]$ (c) $f(x) = \frac{e^{3x}}{1 + e^{6x}}$ on $[\frac{-1}{6} \ln(3), 0]$.
2. Use Maple commands that you type in a Maple worksheet to find the average value of one of the above functions on the given interval.
3. Use the *Volume of Revolution* maplet to visualize the following solids of revolution formed by rotating a region about the x -axis. Also, use the maplet to test your formulation of the definite integral for the volume of the solid, and your evaluation of this integral.
 - (a) $y = x$, $y = 2 - x^2$, $x = 0$, $x = 1$
 - (b) $y = \sin(x)$, $y = \cos(x)$, $x = 0$, $x = \frac{\pi}{4}$
 - (c) $y = x^2$, $y = x^3$
 - (d) $y = \frac{e^{3x}}{\sqrt{1 + e^{6x}}}$, $x = 0$, $x = 1$, $y = 0$
 - (e) $y = 1 - x^2$, $y = 2 + x^2$, $x = -1$, $x = 1$
4. Repeat the previous problem for the following solids of revolution formed by rotating a region about the y -axis.
 - (a) $y = x^2$, $y = x^3$
 - (b) $y = 1/x$, $x = 2$, $y = 2$
 - (c) $y = \sqrt{\frac{1 - x^2}{x^2}}$ ($x > 0$), $x = 0$, $y = 0$, $y = 2$
 - (d) $y = \ln(x)$, $x = 1$, $y = 1$
5. Repeat one example from each of Activity 3 and Activity 4 using the **VolumeOfRevolution** command in a Maple worksheet. Before using the **VolumeOfRevolution** command, enter and execute the command: `with(Student[Calculus1]);`

Additional Notes

- Remember that the **Student[Calculus1]** package must be loaded before the **VolumeOfRevolution** command can be used.
- The first project will be handed out in next week's lab; the lab report will be due one week later. The lab will involve use of the **VolumeOfRevolution** command.