Review of Calculus I

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Overview

The fundamental operations of calculus are limits, derivatives, and integrals (definite and indefinite). The goal of this week's lab is to review those basic operations using Maple.

Maple Essentials

- Maple commands introduced in this lab:

Command	Description
`:=x->`	define a function of x. eg: `f:=x->sqrt(x);  g:=t->2*sin(t);`
`limit`	compute limits or sided limits: `limit(f(x),x=a);` finds the limit, $\lim_{x\to a} f(x)$. `limit(f(x),x=a,right);` finds the one-sided limit from right, $\lim_{x\to a^+} f(x)$. `limit(f(x),x=a,left);` finds the one-sided limit from left, $\lim_{x\to a^-} f(x)$.
`diff`	compute derivatives of an expression: `diff(f(x), x);` finds the derivative of $f(x)$ with respect to x. `diff(f(x), x$n);` finds the nth derivative of $f(x)$.
`int`	compute definite and indefinite integrals: `int(f(x),x);` evaluates the indefinite integral, $\int f(x)\,dx$; `int(f(x),x=a..b);` evaluates the definite integral, $\int_a^b f(x)\,dx$.
`plot`	plot one or more functions on a specified window `plot(f(x), x=a..b);` plots the graph of $y = f(x)$ for $a < x < b$; `plot([f(x),g(x)], x=a..b);` graphs two functions in a single plot
`simplify`	`simplify(f(x));` simplifies $f(x)$

  Note that Maple's **int** command does not include any constants of integration. Whenever you evaluate an indefinite integral, do not forget to include a constant of integration (+C) in your answer.

- Basic Functions and Expressions: Maple uses `sqrt(x)` for $\sqrt{x}$, `abs(x)` for $|x|$, `exp(x)` for e^x, and `x^n` for x^n. Other basic functions can be typed in as what they are but you must carefully use ( ) to group together and match up expressions as needed. You must also type in * when multiplication is presented. For example, you need to type in `sin(x)` instead of `sinx` for $\sin x$ and `2*x/(x+y)` instead of `2x/x+y` for $\frac{2x}{x+y}$. Finally, do not forget to type in ; at the end of a line that is to be executed.
- Palettes: The **Expression** palette can be used instead of typing the full Maple commands for many functions and operations. The **Common Symbols** palette contains other symbols, including π (`Pi`) and ∞ (`infinity`).

Related Course Material/Preparation

Review the basic terminology and properties of limits, derivatives, and integrals.

Activities

Many of the problems below can be solved in Maple using different interface methods: typing in a command directly, right-clicking over the expression, or using palettes. Try and/or combine different methods when possible. You should make it a habit to define a function or assign an expression to a name first. You should always inspect the output to verify that the function or expression was entered correctly. You can then apply operations by right-clicking or referring to the name or the equation label (if you use commands or palettes).

1. Use Maple to compute the derivative of the following functions:

   (i) $f(x) = x^3 \ln(x)$ (ii) $g(x) = \dfrac{\arctan(x)}{\sqrt{x^2+1}}$ (iii) $h(x) = \dfrac{1-\ln(x)}{1+\ln(x)}$

   Notice that if you use a right-click to obtain the derivative the resulting derivative will be defined as a function. If you use the `diff` command the resulting derivative will be an expression.
2. Use Maple to compute the first and 100th derivatives of $f(x) = x\sin(2x)$.
3. Use Maple to evaluate the following indefinite and definite integrals:

   (i) $\displaystyle\int \frac{x+1}{(x-2)^2}\,dx$ (ii) $\displaystyle\int_{-1}^{3} xe^{-x^2}\,dx$ (iii) $\displaystyle\int_0^{\pi/2} \left|\frac{1}{2} - \cos x\right|\,dx$
4. Use Maple to evaluate the following limits:

   (i) $\displaystyle\lim_{x\to 0} \frac{x^2 \arccos(x)}{(\sin(2x))^2}$ (ii) $\displaystyle\lim_{n\to\infty} (1+3/n)^{2n}$ (iii) $\displaystyle\lim_{t\to 1^+} \tan\left(\frac{\pi}{2}t\right)$
5. Let
$$f(x) = \ln\left(\frac{(x^2+2)^2}{\sqrt{4x^2+1}}\right).$$
   In a Maple worksheet:
   (a) Define the function using proper Maple notation.
   (b) Use right-clicking to find the first and second derivatives of $f(x)$. Assign the derivatives to names using `:=` with appropriate label references.
   Hint: *Reasonable names for these quantities would be* `df` *and* `ddf`*.*
   (c) Find $f(1)$, $f'(1)$, and $f''(1)$. Explain what each of these values tells you about the function $f(x)$ when $x = 1$.
   (d) Plot the graph of $y = f'(x)$ on the interval $-4 < x < 4$ and estimate the intervals on which $y = f(x)$ is increasing.
   (e) Plot the graph of $y = f''(x)$ on the interval $-4 < x < 4$ and estimate the intervals on which $y = f(x)$ is concave up.
   (f) Create a single plot containing the graphs of $y = f(x)$, $y = f'(x)$, and $y = f''(x)$ for $-4 < x < 4$. (Don't forget to give your graph a title.) Did you estimate correctly in (d) and (e)?
   (g) Right-click on your graph to see the context menu. Under the option `Legend`, click `Show Legend`. Double click to add appropriate names for each of the curves.

Assignment

Use Maple to do the following exercises from the textbook: 55 on page 266, 10 on page 437, and 100 on page 441. Turn in your answers **and** the Maple worksheet used to obtain them at the beginning of next week's lab.