Graphical Analysis in Polar Coordinates

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Overview

One of the most challenging aspects of polar coordinates is being able to visualize the graph of a polar function, $r = f(\theta)$. An animation showing exactly how the curve is traced out as the angle moves through its domain is even more useful than a static graph of the function.

The simplest polar plots can be created with the `plot` command — with one additional argument. To create an animation in polar coordinates it is easier to work with a *parametric form* of the equation. (Parametric curves will be discussed in more detail in Calculus III.)

Related Course Material/Preparation

- §11.1.
- Know the basic conversions between rectangular and polar coordinates:

$$r = \sqrt{x^2 + y^2} \qquad x = r\cos(\theta)$$
$$\tan\theta = \frac{y}{x} \qquad y = r\sin(\theta)$$

- Remember that all angles need to be specified in radians.
- Be prepared to create some surprising plots that would be almost impossible to create in rectangular coordinates.

Maple Essentials

- The *PolarCurveID* and *Basic14Polar* maplets are available from the course website:

  `http://www.math.sc.edu/calclab/142L-F07/labs`

- New Maple commands introduced in this lab include:

Command	Description
`arctan( y, x )`	two-argument version of the inverse tangent this is essentially equivalent to `arctan(y/x)` except that the signs of `x` and `y` are used to extend the range from $(-\frac{\pi}{2}, \frac{\pi}{2})$ to $(-\pi, \pi)$; this modification makes the two-argument arctan ideal for converting from rectangular to polar coordinates
`plot( ..., coords=polar);`	plot a function in polar coordinates the most common usage is: > `R :=t-> 2*cos(4*t)` > `plot( R(t), t=0..2*Pi, coords=polar );`
`animatecurve`	animated sketch of a curve e.g., the limaçon $r = 1 + 3\sin(\theta)$ could be animated as follows: > `R :=t-> 1 + 3*sin( t );` > `animatecurve([R(t),t,t=0..2*Pi], coords=polar);` **Note:** *Execute* `with( plots ):` *before using* `animatecurve`.
`unassign`	remove assignments from a Maple name to prevent the name from evaluating to its value, it is necessary to enclose each name in single quotes, e.g., > `unassign( 'x', 'y', 'r' );`

Activities

1. Convert the following points to polar coordinates: (2,0), (3,3), (0,2), (-2,3), (-2,-5), (0,-3), (1,$-\sqrt{3}$). **Note:** *Compare the angles obtained with* `arctan(y/x)` *and* `arctan(y,x)`.

   Example: Find the polar coordinates of the point $(-2, 2\sqrt{3})$.
   ```
   > a := -2;
   > b := 2*sqrt(3);
   > theta := arctan(b, a);
   > r := sqrt(a^2+b^2);
   > [r, theta];
   ```
2. For each of the curves below:
   - Find a parameter interval that traces the curve exactly once. See the steps below.
   - Plot the curve in polar coordinates.
   - Animate the sketching of the curve.
     Hint: *A polar function $r = f(\theta)$ can be written in parametric form as $r = f(t)$, $\theta = t$.*
     Note: *Optional arguments to the* `animatecurve` *command include:*
     – `numpoints=`*num instructs Maple to use num points in each frame of an animation; the default number of points is 50.*

(i)	$r = 2 + \sin(\theta)$	(ii)	$r = \cos(4\theta)$	(iii)	$r = 3(1 - \cos(\theta))$
(iv)	$r = \sin\left(\frac{\theta}{5}\right)$	(v)	$r = \sin(\theta) + \cos\left(\frac{\theta}{3}\right)$	(vi)	$r = 2 + \sin\left(\frac{5\theta}{3}\right)$
(vii)	$r = \ln(\theta)$	(viii)	$r = \frac{\theta}{2}$	(ix)	$r = 1 + (\cos(\theta))^3$
(x)	$r = (\cos(\theta))^2$	(xi)	$r^2 = \cos(2\theta)$		

3. The polar function $r = e^{\cos(\theta)} - 2\cos(4\theta) + \left(\sin\left(\frac{\theta}{4}\right)\right)^3$ is called the "butterfly curve".
   (a) Find a parameter interval that traces this curve exactly once.
   (b) Plot or animate the curve.

Finding the parameter interval.

1. Use Maple to evaluate the curve with $\theta = \theta + 2n\pi$.
2. Find the smallest n such that the old curve and the new curve are equivalent. That is, so the second term of your argument is an even multiple of π.
3. Graph the curve with period $2n\pi$ using the n that you just found.
4. If the curve is traced twice, reduce the period by half.

Example: Activity 2(i)
```
> r := theta -> 2 + sin(theta);
> eval(r(theta), theta = theta + 2*n*Pi);
```
Note: *The resulting curve is periodic with $n = 1$, so we graph from 0 to 2π.*
```
> plot(2 + sin(theta), theta=0..2*Pi, coords=polar);
> animatecurve( [2 + sin(theta), theta, theta=0..2*Pi], coords=polar );
```

Assignment

- There is no assignment this week but you need to complete an end-of-course survey. Your TA has instructions for turning in the survey.
- You have just completed the last Calculus II lab. Congratulations and have a great break!