Integration Methods: by Substitution and by Parts

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Overview

The objective of this lab is to provide some Maple tools to help you practice integration skills, such as integration by substitution and integration by parts.

Maple Essentials

- Important Maple commands introduced in this lab:

Command/Example	Description

`Hint(Expression);`
 Examples:
 `Hint(Int(f(x),x));`
 Note: *You need to load the* `Student[Calculus1]` *package before using this command.*
	returns `Hints` for a calculus problem matching the `Expression` which is typically in form of `Diff(...)`, `Int(...)`, or `Limit(...)`.
	 Note: *The first letter of* `Hint, Diff, Int, and Limit` *must be capitalized.*

`Rule[rule](Expression);`
 Examples:
 `Rule[parts,U,V](Int(f(x),x));`
 `Rule[h1](Int(f(x),x));`
 `Rule[h1[2]](Int(f(x),x));`
	used to apply a `rule` obtained from `Hint` to a matching problem.
	 Note: *It is often easier to use names and equation labels as inputs. In the last two examples, we assign our hint to* `h1`.

Maple is very capable when evaluating integrals, derivatives, and limits, but if you want to see individual steps or methods to be applied for such calculus problems (rather than just final answers), you need to use commands `Hint` and `Rule`. However, there are a few things that you should know:

1. There are many different ways to do a calculus problem and Maple's way may be different from yours. While Maple in general will give good hints, you may find a different or better way to do the problem.

2. Because of the constant C in integral formulas, your final answer to an indefinite integral may look different from the one given by Maple. The best way to be sure that the both are correct is to take the derivative of the difference to see if it equals zero.

3. Maple may use methods that we have not learned yet.

4. Maple should be used as a tool to facilitate your learning. Remember, you will need to be able to evaluate these integrals by hand. Do not rely too heavily on Maple.

- The *Integration by Substitution* maplet is available from the course website:

  http://www.math.sc.edu/calclab/142L-F07/labs → <u>Integration by Substitution</u>

This maplet is designed to help you practice the steps involved in integration by substitution. Click on **Indefinite Integral** or **Definite Integral** to have the maplet generate a problem for you to practice. For a definite integral, you will be asked to calculate the associated indefinite integral first. You can then choose to change the limits of integration or substitute to return to the original variable. You can click **Hint** or **Show** if you need help or want to skip a step.

- The *Integration by Parts* maplet is available from the course website:

  http://www.math.sc.edu/calclab/142L-F07/labs → <u>Integration by Parts</u>

  This maplet is designed to help you practice the steps involved in integration by parts. Its interface is very similar to the last maplet.

Related course material

§6.3 and §8.2 of the textbook.

Activities

1. Launch *Integration by Substitution* maplet from the course web page and do a few practice problems.

2. Launch *Integration by Parts* maplet from the course web page and do a few practice problems.

3. Use Maple to help you to evaluate the following:

   (a) $\displaystyle\int \sin x \cos^6 x \ dx$

   (b) $\displaystyle\int x^5 \ln x \ dx$

   (c) $\displaystyle\int x \sin x \ dx$

   (d) $\displaystyle\int \frac{x^2 + 1}{\sqrt{x^3 + 3x + 1}} \ dx$

   (e) $\displaystyle\int \sin^{-1} x \ dx$

   (f) $\displaystyle\int x e^x \ dx$

Example Problem

Evaluate $\displaystyle\int x^3 e^{x^2} \ dx$. (Exercise 27 on page 520)

1. Start a Maple session and don't forget to load `Student[Calculus1]` package.

   > `with(Student[Calculus1]):`

2. Assign expression to be integrated as a function $f(x)$.

   > `f:=x -> x^3*exp(x^2);`

3. Apply `Hint` and assign the hint to `h1`.

   > `h1:=Hint(Int(f(x),x));`

   The return suggests to use parts with $U = x^2$ and $V = e^{x^2}/2$ or a substitution with $U = x^2$. Notice that $U = x^3$ won't work for parts since $V = \int e^{x^2} dx$ cannot be evaluated.

4. To apply the first hint, enter the following:

   > `Rule[h1](Int(f(x),x));`

5. To apply the second hint, enter the following:

   > `Rule[h1[2]](Int(f(x),x));`

6. You should be able to complete the problem by hand after seeing the hint applied, but you may continue to receive and apply hints using the equation labels if you need to.

Assignment

Complete lab activities and review Labs A-E for Hour Quiz 1 next week.