

# Project 1: Goblet Design

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## Instructions

The goal of this project is to design the most visually appealing goblet that meets the following criteria:

- the goblet will be molded using a symmetric mold, that is, the goblet must be a solid of revolution;
- the goblet must hold between  $12\text{in}^3$  and  $14\text{in}^3$  of liquid and use less than  $8\text{in}^3$  of glass;
- the height of the center of mass must be less than 3 times the radius of the foot so the goblet will be reasonably stable;
- thickness of the glass must be at least 0.08 in at its thinnest point.

Your report is due at the beginning of the next lab meeting and should follow the guidelines set forth in the *What is a Project Report?* document on our lab web page. In particular, your report should include the following:

- a detailed description of your design.
- a (2-D) plot of the region to be revolved and a (3-D) plot of the goblet
- detailed numerical results showing that the criteria are satisfied.

## Maple Essentials

Maple commands introduced for this project:

Command	Description
<code>piecewise</code>	define a piecewise-defined function The general syntax to represent $\begin{cases} f_1, & \text{cond}_1 \\ f_2, & \text{cond}_2 \\ \vdots & \vdots \\ f_n, & \text{cond}_n \end{cases}$ is: <code>piecewise( cond<sub>1</sub>, f<sub>1</sub>, cond<sub>2</sub>, f<sub>2</sub>, ..., cond<sub>n</sub>, f<sub>n</sub> );</code> where each $\text{cond}_i$ is an inequality and each $f_i$ is an expression.
<code>solve</code>	solve an equation or system of equations <code>solve( eqn, var );</code> solves an equation, $\text{eqn}$ , for one variable, $\text{var}$ . <code>solve( {eqn1, eqn2}, {var1, var2} );</code> solves a system of two equations for two variables.
<code>assign</code>	<code>assign( values );</code> assigns the values returned by the <code>solve</code> command

## An Example in Maple:

Most maple commands used here were introduced in past labs. For those of you who have taken Maple in a Calculus I class here at USC, you may want to look back at the Roller Coaster project. Also, you may want to review the use of the `VolumeOfRevolution` command from last week's lab. For your convenience, this example is available in a Maple worksheet from our lab web page.

1. Basic Analyses: By studying our favorite goblets at home, we decide that our new goblet should have a foot (base) of 1.4 inches in radius and 0.5 inches in height. We would also like to have a slightly curved stem of 6 inches long and about 0.2 inches in radius. Finally, we want it to have a bowl that gradually extends to a radius of 1 inch in the first inch from the stem, then to a radius of 0.9 inches in the next 0.5 inches, and to a radius of 1.6 inches in the last 1.5 inches. Put it in sideways and our goblet hence extends from  $x = 0$  to  $x = 9.5$ . The outside profile of our goblet can therefore be described by a piecewise function that consists of three functions  $f_1(x)$ ,  $f_2(x)$ , and  $f_3(x)$  (corresponding to base, stem, and bowl sections of our goblet) defined over  $0 \leq x < 0.5$ ,  $0.5 \leq x < 6.5$ , and  $6.5 \leq x \leq 9.5$ , respectively. You may want to use more than three functions for your design.

## 2. Working with Maple

- (a) First, load needed packages using the `with` command.
- ```
> with(plots):
> with(Student[Calculus1]):
```
- (b) For the foot, we want to use a parabola that starts at  $(0, 1.4)$  and ends at  $(0.5, 0.2)$ . You can use Maple to find this, but the calculation is not difficult by hand. The following parabola will do the job.
- ```
> f1:=x->1.4-4.8*x^2;
```
- (c) For a plain stem of radius 0.2, we may just set  $f2(x) = 0.2$ . To have a slightly curved stem, we will add a slight wave to it using the sine function. For this particular goblet, we take the amplitude 0.05 as the height of the wave and shift the function 0.5 to the right so we have  $f2(0.5) = f1(0.5) = 0.2$ .
- ```
> f2:=x->0.2+0.05*sin(x-0.5);
```
- (d) By our design, our bowl needs to satisfy the following four conditions: It connects the stem at  $x = 6.5$  and takes values 1, 0.9, and 1.6 at  $x = 7.5$ ,  $x = 8$ , and  $x = 9.5$ , respectively. Therefore, let us define  $f3(x) = ax^3 + bx^2 + cx + d$ , where  $a, b, c, d$  are four constants to be solved later to satisfy the above four requirements. Here are  $f3(x)$  and 4 equations.
- ```
> f3:=x->a*x^3+b*x^2+c*x+d;
> eq1:=f3(6.5)=f2(6.5);
> eq2:=f3(7.5)=1;
> eq3:=f3(8)=0.9;
> eq4:=f3(9.5)=1.6;
```
- (e) Now, let's solve for  $a, b, c$ , and  $d$  using `solve` command and assign solutions to *values*.
- ```
> values:=solve({eq1,eq2,eq3,eq4},{a,b,c,d});
```
- (f) We can then plug these solved values into our functions using the `assign` command.
- ```
> assign(values);
```
- (g) We are now ready to put all three functions together as a piecewise function and plot it to see the outside profile.
- ```
> F:=x->piecewise(x<0.5,f1(x),x>=0.5 and x<6.5,f2(x),x>=6.5,f3(x));
> plot(F(x),x=0..9.5,y=0..4,scaling=constrained);
```
- Note:** The `scaling=constrained` insures that one unit on the  $x$ -axis equals one unit on the  $y$ -axis.
- (h) To make sure that the thickness of the glass is at least 0.08 inches, we take the bowl part of the inside profile to be 0.1 inches less than the bowl part of the outside profile as follows:
- ```
> g3:=x->f3(x)-0.1;
> G:=x->piecewise(x<=6.5,0,x>6.5,g3(x));
> plot(G(x),x=0..9.5,y=0..4,scaling=constrained);
```
- (i) Here is the goblet. Rotate and/or right click the plot to try some visual effect options!
- ```
> VolumeOfRevolution(F(x),G(x),x=0..9.5,output=plot,orientation=[0,180],
title='Example');
```
- (j) We still need to check the criteria. For the center of mass, we can see that it is located on the  $x$ -axis, say, at  $x = CM$ . It is not too hard to derive the following formula for  $CM$ :

$$CM = \frac{\int_0^{9.5} x(F^2(x) - G^2(x)) dx}{\int_0^{9.5} (F^2(x) - G^2(x)) dx}.$$

- ```
> Capacity:=evalf(VolumeOfRevolution(G(x),0,x=0..9.5,output=integral));
> Glass:=evalf(VolumeOfRevolution(F(x),G(x),x=0..9.5,output=integral));
> CM:=evalf(int(x*(F(x))^2-(G(x))^2,x=0..9.5)/int((F(x))^2-(G(x))^2,x=0..9.5));
> Ratio:=CM/F(0);
```

3. Conclusion: Looks like we have a stable goblet that satisfies the requirement on amount of glass used. However, the goblet does not hold enough. The visual appearance of the goblet can also be improved. Now, it is up to you to design a more interesting and better looking goblet that satisfies all the criteria.