

Review of Calculus I

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Overview

The fundamental operations of calculus are limits, derivatives, and integrals (definite and indefinite). The goal of this week's lab is to review those basic operations using Maple.

Maple Essentials

- Maple commands introduced in this lab:

Command	Description
<code>:=x-></code>	define a function of x . eg: <code>f:=x->sqrt(x);</code> <code>g:=x->2*sin(x);</code>
<code>limit</code>	compute limits or sided limits: <code>limit(f(x), x=a);</code> finds the limit, $\lim_{x \rightarrow a} f(x)$. <code>limit(f(x), x=a, right);</code> finds the one-sided limit from right, $\lim_{x \rightarrow a^+} f(x)$. <code>limit(f(x), x=a, left);</code> finds the one-sided limit from left, $\lim_{x \rightarrow a^-} f(x)$.
<code>diff</code>	compute derivatives of an expression: <code>diff(f(x), x);</code> finds the derivative of $f(x)$ with respect to x . <code>diff(f(x), x\$n);</code> finds the n th derivative of $f(x)$.
<code>int</code>	compute definite and indefinite integrals: <code>int(f(x), x);</code> evaluates the indefinite integral, $\int f(x) dx$; <code>int(f(x), x=a..b);</code> evaluates the definite integral, $\int_a^b f(x) dx$.
<code>plot</code>	plot one or more functions on a specified window <code>plot(f(x), x=a..b);</code> plots the graph of $y = f(x)$ for $a < x < b$; <code>plot([f(x), g(x)], x=a..b);</code> graphs two functions in a single plot

Note that Maple's `int` command does not include any constants of integration. Whenever you evaluate an indefinite integral, do not forget to include a constant of integration (+C) in your answer.

- Basic Functions and Expressions: Maple uses `sqrt(x)` for \sqrt{x} , `abs(x)` for $|x|$, `exp(x)` for e^x , and `x^n` for x^n . Other basic functions can be typed in as what they are but you must carefully use `()` to group together and match up expressions as needed. You must also type in `*` when multiplication is presented. For example, you need to type in `sin(x)` instead of `sindx` for $\sin x$ and `2*x/(x+y)` instead of `2x/x+y` for $\frac{2x}{x+y}$. Finally, do not forget to type in `;` at the end of a line that is to be executed.
- Palettes: The **Expression** palette can be used instead of typing the full Maple commands for many functions and operations. The **Common Symbols** palette contains other symbols, including π (Pi) and ∞ (infinity).

Related Course Material/Preparation

Review the basic terminology and properties of limits, derivatives, and integrals.

Activities

Many of the problems below can be solved in Maple using different interface methods: typing in a command directly, right-clicking over the expression, or using palettes. Try and/or combine different methods when possible. You should make it a habit to define a function or assign an expression to a name first. You should always inspect the output to verify that the function or expression was entered correctly. You can then apply operations by right-clicking or referring to the name or the equation label (if you use commands or palettes).

1. Use Maple to compute the derivative of the following functions:

$$(i) \quad f(x) = x^3 \ln(x) \qquad (ii) \quad g(x) = \frac{\arctan(x)}{\sqrt{x^2 + 1}} \qquad (iii) \quad h(x) = \frac{1 - \ln(x)}{1 + \ln(x)}$$

2. Use Maple to compute the first and 100th derivatives of $f(x) = x \sin(2x)$.

3. Use Maple to evaluate the following indefinite and definite integrals:

$$(i) \quad \int \frac{x+1}{(x-2)^2} dx \qquad (ii) \quad \int_{-1}^3 x e^{-x^2} dx \qquad (iii) \quad \int_0^{\pi/2} \left| \frac{1}{2} - \cos x \right| dx$$

4. Use Maple to evaluate the following limits:

$$(i) \quad \lim_{x \rightarrow 0} \frac{x^2 \arccos(x)}{(\sin(2x))^2} \qquad (ii) \quad \lim_{n \rightarrow \infty} (1 + 3/n)^{2n} \qquad (iii) \quad \lim_{t \rightarrow 1^+} \tan\left(\frac{\pi}{2}t\right)$$

5. Let

$$f(x) = \ln\left(\frac{(x^2 + 2)^2}{\sqrt{4x^2 + 1}(x^2 + 1)}\right).$$

In a Maple worksheet:

- define the function and then find $f(10)$ and $f(t^2)$.
- find and name the first and second derivatives of $f(x)$
Hint: Reasonable names for these quantities would be df and ddf .
- plot the graph of $y = f''(x)$ on the interval $-5 < x < 5$ and estimate the intervals on which $y = f(x)$ is concave up.
- create a single plot containing the graphs of $y = f(x)$ and $y = f'(x)$ for $-5 < x < 5$ and verify that $f'(x) = 0$ at each critical number of $y = f(x)$. Did you estimate correctly in (c)?
- Right-click on your graph to see the context menu. Under the option **Legend**, click **Show Legend**. Now, from the same context menu, click **Edit Legend**. Enter an appropriate label for the first curve. Repeat this step for the other curve in the plot.

Assignment

Use Maple to do the following exercises from the textbook: 55 on page 266, 17 on page 437, and 99 on page 441. Turn in your answers **and** the Maple worksheet used to obtain them at the beginning of next week's lab.