Series: Convergence Tests

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Overview

The intent of this lab is to introduce a maplet to provide additional practice determining the convergence or divergence of series.

Maple Essentials

- A link to the *SeriesConvergenceTestDrill* maplet can be found on the course website (last column in Lab 11):

http://www.math.sc.edu/calclab/142L-F06/labs/

The first hurdle in determining the convergence or divergence of a series is to select an applicable test. Then there are detailed steps in each test to be carried out and some of them could easily be overlooked. The best way (and the only way) to overcome those difficulties is to have a lot of practice and this maplet is made for that. You can input your own series or the maplet will randomly generate one for you to practise. To get numerical evidence, you can then choose a range of indices and plot terms and/or partial sum. Of course, you may go directly to Step C to select an applicable test in best of your knowledge (you can always try another one if it turns out otherwise.) It then opens up a new window and shows you all steps that need to be worked out. If you need further reminder of the test that you are using, click `Hind`. This is also a great tool to check your work and answers for homework problems (but don't depends on it too much). Try to use it to help you to understand so you will be able to do problems on your own eventually.

Preparation

§10.4, §10.5, and §10.6. Be sure to review steps and to understand conditions needed so that a particular test can be applied.

Assignment

Exercises 15-20 on page 715.

Activities

For each of the following series, decide first which test should be used in determining whether the series diverges or converges and then use *SeriesConvergenceTestDrill* maplet to carry out detailed steps. Try another test if it is not applicable or the answer is inconclusive.

(1) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k+1}}$

(2) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$

(3) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\ln(k+1)}$

(4) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k\sqrt{k^2+1}}$

(5) $\displaystyle\sum_{k=1}^{\infty} \frac{k+1}{k!}$

(6) $\displaystyle\sum_{k=1}^{\infty} \frac{(-3)^k}{k!}$

(7) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$

(8) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k(k^2+1)}{2k^2+k-1}$

(9) $\displaystyle\sum_{k=1}^{\infty} \frac{2^k 3^k}{k^k}$

(10) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+1)(k+2)}}$

(11) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(3k-2)^{k+0.5}}$

(12) $\displaystyle\sum_{k=1}^{\infty} \frac{tan^{-1}k}{k^2+1}$

(13) $\displaystyle\sum_{k=1}^{\infty} \frac{\ln(k!)}{k^3}$

(14) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k 3^k k!}{(2k)!}$

(15) $\displaystyle\sum_{k=1}^{\infty} (-1)^k \left(\frac{k}{k+1}\right)^k$