

# Implicit Differentiation

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## Overview

This lab provides experience working with functions defined implicitly.

## Maple Essentials

- Important Maple commands introduced in this lab are:

Command	Description	Example
<code>display</code>	display plots in a single plot (need <code>plots</code> package)	<code>display([F,G],title='Fig1');</code>
<code>implicitplot</code>	create graph of function defined implicitly (need <code>plots</code> package)	<code>implicitplot(x*y=1,x=0..1,y=0..1);</code>
<code>pointplot</code>	plot points (need <code>plots</code> package)	<code>pointplot([1,2], color=red, symbolsize=18);</code>
<code>implicitdiff</code>	compute derivatives of functions defined implicitly	<code>implicitdiff(f,y,x);</code> <code>implicitdiff(f,y,x\$2);</code>
<code>fsolve</code>	compute a solution of equations numerically	<code>fsolve({f=1,g=x^2},{x,y});</code> <code>fsolve({f,g},{x,y},{x=0..1,y=0..2});</code>
<code>with</code>	load a Maple package	<code>with(plots):</code> <code>with(plots);</code>

- The *ImplicitDifferentiation* maplet is available from the course website:

<http://people.math.sc.edu/calclab/141L-S19/labs> → ImplicitDifferentiation

## Related course material/Preparation

§3.7

## Assignment

Complete lab activities and your lab instructor will give other assignment for each section.

Hint for using `implicitplot`: Start with a big range for both  $x$  and  $y$  in `implicitplot` to see the size of the view window the graph will display and then re-plot the graph with that view window for a better plot.

## Activities

**Problem 1:** Find the equation of the tangent line to the curve  $2(x^2 + y^2)^2 = 25(x^2 - y^2)$  at the point  $(3, 1)$ . Then graph the curve, the point, and the tangent line with a viewing window of  $(-5, 5) \times (-2, 4)$ .

**Problem 2:** Find all points where the tangent line to the graph of  $x^2y - xy^2 = 2$  is horizontal or vertical. (Hint: The tangent line is vertical where  $dx/dy = 0$ .)

**Problem 3:** Find  $d^2y/dx^2$  and  $d^3y/dx^3$  if  $y$  is defined implicitly by  $y + \sin y = x$ .

### Example Problem

- a) Use implicit differentiation to find  $dy/dx$  for the Folium of Descartes  $x^3 + y^3 = 3xy$ .
- b) Find the equation of the tangent line to the Folium of Descartes at the point  $(3/2, 3/2)$ .
- c) Graph the curve, the point, and the tangent with a viewing window of  $(-3,3) \times (-4,3)$ .
- d) At what point(s) in the first quadrant is the tangent line to the Folium of Descartes horizontal?

Steps:

1. Start a Maple session with `restart;` and load the Maple `plots` package. This package allows us to plot points, use the display command, use the commands for implicitly-defined functions, and more. Notice that we used `:` instead of `;`. The difference is that the maple does not display the output with `:`.
 

```
> restart;
> with(plots):
```
2. For part a), simply assign the Folium of Descartes to, say, `FD`, then use command `implicitdiff` to find  $dy/dx$ .
 

```
> FD:=x^3 +y^3 =3*x*y;
> dydx:=implicitdiff(FD,y,x);
```

 (Notice that `implicitdiff(f,x,y)`; computes  $dx/dy$  and `implicitdiff(f,y,x$n)`; computes  $d^n y/d^n x$ . You will need them to do problem 2 and problem 3, respectively.)
3. Next, to find the tangent line, we need a point and a slope. The point  $(3/2, 3/2)$  is given and we find the slope `m` by evaluating  $dy/dx$  at this point.
 

```
> m:= eval(dydx, {x=3/2, y=3/2});
```
4. Find the equation of the tangent line by the point-slope formula  $y = m(x - x_1) + y_1$ .
 

```
> L:=x-> m*(x-3/2)+3/2;
```
5. Next, write (and assign) commands to plot the curve, the point, and the tangent line. Write the commands separately using `:` so Maple does not display the output yet. (In the first plot command, the option `numpoints=10000` will insure a smooth curve.)
 

```
> P1:= implicitplot(FD, x=-3..3, y=-4..3, numpoints=10000):
> P2:= pointplot([3/2,3/2], color=green, symbolsize=15):
> P3:= plot(L(x), x=-3..3, y=-4..3, color=blue, linestyle=DOT):
```
6. These plots can then be displayed on a single plot using the `display` command.
 

```
> display([P1, P2, P3], title='Figure 1');
```
7. From the graph, we can see that the answer to part d) is a point located approximately at  $(1.2, 1.5)$ . Since this point is on the curve and the  $dy/dx = 0$  at this point, we can find it's location by solving those two equations.
 

```
> fsolve({FD,dydx=0},{x,y},{x=1..2,y=1..2});
```

 (For a numerical solution in a specified region, `fsolve` in general does a better job than `solve`.)

### Additional Notes

The ImplicitDifferentiation maplet provides additional practice finding the slope of a curve at a point.