Differentiation and Tangent Lines

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Overview

We will learn in this lab how to use Maple to find derivatives and the equation of the tangent line to a curve at a given point.

Maple Essentials

- Important Maple commands introduced in this lab are:

Command	Description	Example
`diff(f(x),x);`	find derivative of f(x)	`diff(k^3*x^2,x);diff(k^3*x^2,k);`
`diff(f(x),x$n);`	find nth derivative of f(x)	`diff(x^8,x$4);diff(x*cos(x),x$100);`

- The *Tangents* tutor is started from Maple interface under the tools menu:
  - **Tools** → **Tutors** → **Calculus - Single Variable** → **Tangents** ...
- The *TangentLine* maplet is started from the course website:
  - **http://people.math.sc.edu/calclab/141L-S19/labs/** → TangentLine

Related course material/Preparation

§3.1 and §3.2. Recall the point-slope form of the equation of the line:

$$y - y_1 = m(x - x_1),$$

where (x_1, y_1) is a point on the line and m is the slope of the line. Next, since point $(x_1, f(x_1))$ is on the tangent line, we can substitute $y_1 = f(x_1)$ and move it to the other side. We hence get:

$$y = m(x - x_1) + f(x_1).$$

Finally, we know that the derivative evaluated at x_1 is the same as the slope of the tangent line to the graph of $y = f(x)$ at x_1. Thus we get the following formula for the equation of the tangent line to the graph of $y = f(x)$ at x_1:

$$y = f'(x_1)(x - x_1) + f(x_1).$$

Activities

A) We already knew how to find the derivative from the definition, that is, use Maple to find the limit

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

A more direct way is to use the command `diff`. To compute the derivative of f(x) with respect to x, you simply type `diff(f(x),x);` (or use the right-clicking). It can also be used to find higher order derivatives. For example, to find the third derivative of x^4, you simply type `diff(x^4,x$3);`. Try the following sets of examples (please pay attention to subtle differences):

1. `diff(k*x^4,x); diff(k*x^4,k); diff(k*x^4,x$4); diff(k*x^4,x$5);`
2. `f:=x->x^4; diff(f,x); diff(f(x),x); diff(f(sin(x)),x);`
3. `diff(sin(x)*cos(x),x$100); diff(x^x*cos(x),x$8);`

B) Find the equation of the tangent line to the graph of $f(x)$ at the point $(x_1, f(x_1))$ for the following functions. Graph the function and its tangent line on the same axes.

1. $f(x) = x^2, x_1 = 1$
2. $f(x) = 2^x, x_1 = 1$
3. $f(x) = \cos(x), x_1 = \frac{\pi}{4}$ (Recall that, in Maple, you type Pi for π)

Example Problem

We will do an example together for $f(x) = x^3 - 2x + 1$ at $x_1 = 2$ in two different ways:

The first way:

1. Launch the *Tangents* tutor.
2. Enter the function as x^3-2*x+1 and x=2, and change the number of iterations to 5.
3. Click **Display**. The tutor will display the function and a series of secant lines, including the tangent line. The equation of the tangent line is displayed on the right.
4. Press the **Animate** button. The tutor will show the progression through the secant lines as Δx gets smaller.
5. The tutor will return the last graph when you click **Close**.
6. If you want to graph the function and the tangent line, assign both in a Maple worksheet and write a plot command.

The second way:

1. Define the function and assign it to f.
   `> f := x -> x^3-2*x+1;`
2. Right-click and choose *Differentiate*. Then use a label (or right-click again) to assign this new function to *df*.
   `> df := label;`
3. Find $f'(2)$ and assign that value to m.
   `> m := df(2);`
4. Find the equation of the tangent line $y = f'(2)(x - 2) + f(2)$ and assign it as a function to L.
   `> L :=x-> m*(x-2)+f(2);`
5. Plot the function and the tangent line using different linestyles.
   `> plot( [f(x), L(x)], x=-1..3, linestyle=[solid, dash]);`

C) The *TangentLine* maplet is a great tool to practice finding the equation of the tangent line by hand. Launch the maplet and click **New Function**. Follow the prompts step by step to find $f(a)$, $f'(x)$, and $f'(a)$. Then enter the equation of the tangent line as follows:

$$y = f'(a)(x - a) + f(a).$$

The maplet will check each of your answers (or on Show if you don't know the answer) and let you know whether you are correct. Please try a few problems to make sure that you really understand how to find the equation of the tangent line by hand.

Assignment

Complete lab activities and your lab instructor will give other assignment for each section.