Tile Design: An Application of Integration

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Overview

This lab will help to develop your understanding of the definite integral as area for non-negative functions. In this lab you will explore how to use functions to create interesting patterns for floor tiles, as well as analyze how much area is represented by various portions of the pattern.

Maple Essentials

New Maple commands introduced in this lab include:

Command	Description
`int(f(x), x=a..b);`	evaluates $\int_a^b f(x)dx$

Preparation

§6.1 introduces the idea of calculating the area under the graph of a non-negative function.
In §6.5, we discover that:

- If the function $f(x) \geq 0$ is continuous on the interval $[a, b]$, then the area between the curve $y = f(x)$ and the x-axis ($y = 0$) over the interval $[a, b]$ is defined by

$$A = \int_a^b f(x)\ dx.$$

More generally, if we want to find the area between two functions $f(x) \geq g(x)$ (both continuous on $[a, b]$) we would do the following:

$$A = \int_a^b f(x)\ dx - \int_a^b g(x)\ dx = \int_a^b (f(x) - g(x))\ dx.$$

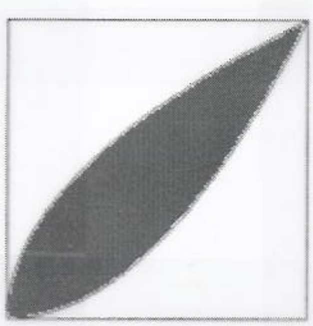
(This will be explained in more detail in §7.1.)

Assignment

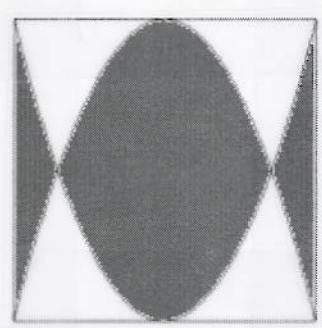
This week's assignment is to complete **Project 2** and prepare a neat and complete project report.

Activities

You have been hired by Columbia's Tile and Flooring to design several new patterns of floor tile. Your contract calls for you to analyze two tile patterns that the company has recently developed. The foremost concern of the company is cost, and this is based on the amount of color required to tint regions in the various tile patterns. Thus, your task is to discover functions to recreate the tile designs below. Once created, you are to determine the area of the darker regions. (It is easiest to assume that each tile is one square foot, thus you can work with functions on the window $[0, 1] \times [0, 1]$.)



Tile A



Tile B

Tile A

The calculations for Tile A are straightforward.

1. Based on our knowledge of functions, we can easily see that the design on Tile A was created using the functions $y = \sqrt{x}$ and $y = x^2$. So we input these in Maple as $f1(x)$ and $f2(x)$.
```
> f1:= x -> sqrt(x);
> f2:= x -> x^2;
```
2. Use the following plot command to confirm our design and view the tile.
```
> plot([f1(x),f2(x)],x=0..1,y=0..1,axes=boxed,tickmarks=[0,0],color=black);
```
3. Next notice that the area tinted with the darker color is exactly the area between the curves over the interval $[0, 1]$. We can calculate this area using a definite integral.
```
> Dark:= int(f1(x)-f2(x), x=0..1);
```
4. Since the entire square is 1 ft^2, we get that the darker region is $\frac{1}{3}\text{ft}^2$ and represents 33.333% of the tile.

Tile B

The calculations for Tile B are a bit more involved.

1. Based on our knowledge of functions, we can easily see that the design on Tile B was created using two parabolas, so we will input generic quadratic equations in Maple as $f1(x)$ and $f2(x)$.
```
> f1:= x -> a*x^2+b*x+c;
> f2:= x -> d*x^2+e*x+f;
```
2. Notice that $f1(x)$ passes through the points $(0, 0)$, $(0.5, 1)$ and $(1, 0)$. We will use this information to create and solve a system of equations to determine the unknown constants in $f1(x)$.
```
> eq1:= f1(0)=0;
> eq2:= f1(0.5)=1;
> eq3:= f1(1)=0;
> f1values:=solve({eq1,eq2,eq3}, {a,b,c});
> assign(f1values);
> f1(x);
```
3. Similarly, notice that $f2(x)$ passes through the points $(0, 1)$, $(0.5, 0)$ and $(1, 1)$. We will use this information to create and solve a system of equations to determine the unknown constants in $f2(x)$.
```
> eq4:= f2(0)=1;
> eq5:= f2(0.5)=0;
> eq6:= f2(1)=1;
> f2values:=solve({eq4,eq5,eq6}, {d,e,f});
> assign(f2values);
> f2(x);
```
4. Use the following plot command to confirm our design and view the tile.
```
> plot([f1(x),f2(x)],x=0..1,y=0..1,axes=boxed,tickmarks=[0,0],color=black);
```
5. Notice that the area tinted with the darker color is in three regions. To find the area of these regions, we will need to first find the two intersection points and assign them to **x1** and **x2**, respectively.
```
> fsolve(f1(x)=f2(x), x);
```
Note: *If Maple does not solve for all of the intersection points at once, you may need to specify an interval.*
6. Once we have the intersection points, we find the area as follows:
```
> DarkA:= int(f2(x)-f1(x), x=0..x1);
> DarkB:= int(f1(x)-f2(x), x=x1..x2);
> DarkC:= int(f2(x)-f1(x), x=x2..1);
> Dark:= DarkA + DarkB + DarkC;
```
7. Since the entire square is 1 ft^2, we get that the darker region is 0.60948ft^2 and represents 60.948% of the tile.