Project 2: Designer Functions

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For this week's project, we will be creating three designer functions. Your report should contain the following information for each of the three functions:

- An explanation of how the stated conditions lead to a system of equations for the unknown parameters in the function. You should also include the system of equations.
- The solution to the system of equations and the function that meets all of the conditions.
- A plot of the function on an appropriate interval that shows all of the essential qualitative features of the function.

In addition, for the quintic polynomial found in 2, find the following:

- All zeros of the function.
- The intervals where the function is increasing, decreasing, concave up, and concave down.

Be creative in your presentation of these results. If, for example, you use a table, remember that the main text needs to explain what information is contained in the table. All floating point numbers should be reported with an accuracy of at least three digits to the right of the decimal point.

These are the functions you need to create:

(1) Find the cubic polynomial $F(x) = ax^3 + bx^2 + cx + d$ with a relative minimum at (-4,-5) and a relative maximum at (2,4).

(2) Find the quintic polynomial $F(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ that passes through the point (-3,2), has a tangent line at $x = 0$ with slope -1, an inflection point at (2,1) and a local **maximum** at (6,-5).
   - You should find a system of 6 equations and 6 unknowns.

(3) Find the function of the form $F(x) = \frac{a+be^x}{c+de^x}$ with asymptotes $y = -2$, $y = 3$, and $x = \ln(5)$ that is **decreasing** on its natural domain.
   - Think about when a non-rational fractional function has a vertical or horizontal asymptote to discover equations. (You may want to review an old lab.)
   - You will have a system of 3 equations and 4 unknowns. Use the **solve** command to solve the system, plug in the values and simplify. The last unknown should cancel out.
   - Remember to use *discont=true* in your plot command so you will be able to see the discontinuity in your graph.
   - **Hint:** You do not have to find any derivatives to solve this problem.