New Functions from Old

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Overview

There are two objectives in this lab:

- Learn to input and manipulate functions using Maple 9.5.
- Use Maple 9.5 to calculate and simplify combinations of functions.

Maple Essentials

New Maple commands introduced in this lab include:

Command	Description
`eval`	`eval( F, x=a );` evaluates expression F with x replaced by a
`simplify`	`simplify( F );` simplifies expression F

Preparation

Read Section 1.3: *New Functions from Old* in Anton.

Assignment

This week's Mastery Quiz asks you to use Maple to evaluate (and simplify) combinations and compositions of functions. The Activities in this lab will help prepare you to answer the Mastery Quiz questions. The deadline for turning in Mastery Quiz 3 will be announced in lab.

Transferring a Mathematical Statement to Maple

Each of the following examples assumes that you have already assigned the expressions to $f, g,$ or h as necessary using `:=` as demonstrated last week.

Mathematical Statement	Maple Translation
$f(a)$	`> eval( f, x=a );`
$f(ax + b)$	`> eval( f, x=ax+b );`
$f(g(x))$	`> eval( f, x=g );`
$f(g(h(x)))$	`> eval( f, x=eval( g,x=h ));`
$\frac{f(x+h)-f(x)}{h}$	`> (eval( f, x=x+h ) - f) / h;`

To simplify any of the above formulas, include `simplify( formula );`.

Activities

1. Find and simplify formulas for $f + g$, $f - g$, fg, and f/g.
   a. $f(x) = \frac{2}{x}$, $g(x) = \frac{4}{x+4}$
   b. $f(x) = \frac{2}{x+1}$, $g(x) = \frac{x}{x+1}$
   c. $f(x) = 1 + \frac{x}{x+1}$, $g(x) = 2 - \frac{1}{x}$

2. Let $g(x) = 4 - \frac{1}{3x+2}$. Find and simplify each of the following.
     a. $g(5s + 2)$ b. $3g(5x)$ c. $g(g(x))$

3. Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$. Simplify your answer.
     a. $f(x) = 3x^2 - x + 7$ b. $f(x) = \frac{1}{(x+1)^2}$

4. Find and simplify $(f \circ g)(x)$ and $(g \circ f)(x)$.
   a. $f(x) = x + \frac{1}{x}$, $g(x) = \frac{x+1}{x+2}$
   b. $f(x) = \sqrt{2x + 3}$, $g(x) = x^2 + 1$

5. Find and simplify $(f \circ g \circ h)(x)$.
   a. $f(x) = \sqrt{x - 1}$, $g(x) = x^2 + 2$, $h(x) = x + 3$
   b. $f(x) = \frac{2}{x+1}$, $g(x) = \cos x$, $h(x) = \sqrt{x + 3}$

6. Express F as a composition of two functions; that is, find f and g such that $F = f \circ g$. Use Maple to verify the composition.
   Note: *Do not choose the identity $(y = x)$ as a function.*
     a. $F(x) = (x^2 + 1)^{10}$ b. $F(x) = \sin(\sqrt{x})$ c. $F(x) = \frac{\tan x}{1 + \tan x}$

7. Express F as a composition of three functions; that is, find f, g, and h such that $F = f \circ g \circ h$. Use Maple to verify the composition.
   Note: *Do not choose the identity $(y = x)$ as a function.*
     a. $F(x) = 1 - 3^{x^2}$ b. $F(x) = \sqrt{2 + |x|}$ c. $F(x) = \cos^4(\sqrt{x})$