Differentiation and Tangent Lines

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Overview
This week’s lab will provide practice finding locally linear approximations to functions. That is, we will be finding the equation of the tangent line to a curve.

Maple Essentials

- The Tangents tutor is started from the Maple 9.5 user interface under the Tools menu:

  Tools → Tutors → Calculus - Single Variable → Tangents...

- The TangentLine maplet is available from the course website:

  http://www.math.sc.edu/calclab/141L-S06/labs/ → TangentLine

- The Maple commands involved with finding and plotting the tangent line to the graph of a (differentiable) function are:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
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<tbody>
<tr>
<td>:=</td>
<td>assignment</td>
</tr>
<tr>
<td>diff</td>
<td>diff( F, x ); differentiate an expression</td>
</tr>
<tr>
<td>eval</td>
<td>evaluate at a point</td>
</tr>
<tr>
<td>plot</td>
<td>plot one or more expressions</td>
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Preparation
Recall the point-slope form of the equation of the line:

\[ y - y_1 = m(x - x_1) \]

where \((x_1, y_1)\) is a point on the line and \(m\) is the slope of the line. Next, solve the equation for \(y\) and we get:

\[ y = m(x - x_1) + y_1. \]

Now, we use the substitution \(y_1 = f(x_1)\) and this becomes:

\[ y = m(x - x_1) + f(x_1). \]

Finally, we know that the derivative evaluated at \(x_1\) is the same as the slope of the tangent line at \(x_1\). Thus we get the following formula for the equation of the tangent line at \(x_1\):

\[ y = f'(x_1)(x - x_1) + f(x_1). \]

Assignment
This week’s mastery quiz asks you to find and graph the tangent lines for several functions. The Activities in this lab will help you answer the Mastery Quiz questions. The TangentLine maplet provides additional practice finding tangent lines.

Activities
We will find the equation of the tangent line to the graph of \(f(x)\) at the point \((x_1, f(x_1))\) for several different functions. We will then graph the function and its tangent line on the same axes.

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Example Problem
We will solve the following problem together in two different ways:

- Find an equation for the line that is tangent to the graph of the differentiable function \( f(x) = x^3 - 2x + 1 \) at \( x_1 = 2 \). Then graph the curve and this tangent line on the same axes.

The first way:
1. Launch the Tangents tutor.
2. Enter the function as \( x^3 - 2x + 1 \) and the base point as 2.
3. Press the Display button.
4. The maplet will return the graph of the function and the tangent line when you press Close.

The second way:
1. Enter and execute one by one the following Maple commands.
   > f := x^3 - 2*x + 1; Assign the function to f.
   > Df := diff( f, x ); Compute the derivative \( f'(x) \) and assign to Df.
   > m := eval( Df, x=2 ); Find \( f'(2) \) and assign to m.
   > L := m*( x-2 )+eval( f, x=2 ); Find the tangent line \( y=f'(2)(x-2)+f(2) \).
   > plot( [f, L], x=-2..3 ); Plot the function and the tangent line.

Additional Notes
- The \texttt{diff} command can also be used to evaluate higher order derivatives. For example, if we wanted to find the second derivative of \( F \), we would use the command \texttt{diff( F, x, x )}. This notation, however, becomes somewhat tedious for higher order derivatives. The command \texttt{x$n} repeats \texttt{x} successively \texttt{n} times and thus shortens our commands. For example, for the fourth derivative of \( F \), we could write \texttt{diff( F, x$4 )} instead of \texttt{diff(F, x, x, x, x )}.
- Not using a color printer? If you want to distinguish the curves on your graph, add the option \texttt{linestyle=[SOLID, DOT]} to your plot command. This will draw \( f(x) \) with a solid line and the tangent line with dots.