Definite Integrals and Riemann Sums

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Overview

This lab will help to develop your understanding of the definite integral as defined via Riemann sums and as computed via the Fundamental Theorem of Calculus.

Maple Essentials

- The *Riemann Sums* tutor can be started from the Tools menu:

  Tools → Tutors → Calculus - Single Variable → Riemann Sums ...

- New Maple commands introduced in this lab include:

Command	Description
`int`	used for definite and indefinite integrals
`int(f, x);`	evaluates $\int f(x)dx$ **Remember:** Maple will not add +C for you.
`int(f, x=a..b);`	evaluates $\int_a^b f(x)dx$

Preparation

Review the definition of area under a curve and approximations of area (last part of §6.4 in Anton) and the Fundamental Theorems of Calculus (§6.6 in Anton). In particular, you should be able to explain the symbols and meaning of the following two equations:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k^*)\Delta x$$

$$\int_a^b f(x)\,dx = F(b)-F(a) \qquad \text{where } F \text{ is an antiderivative of } f$$

Activities

1. Use the `Riemann Sums` tutor to approximate $\int_2^{10}\frac{1}{x}\,dx$ with the Riemann sum $\sum_{k=1}^{4} f(x_k^*)\Delta x$ where:
   (a) x_k^* is the left endpoint of each subinterval
   (b) x_k^* is the right endpoint of each subinterval
   (c) x_k^* is the midpoint of each subinterval
2. Make a table of approximate values for $\int_2^{10}\frac{1}{x}\,dx$ using left, right, or midpoint approximations (your choice) with n = 4, 8, 16, 32, 64, and 128 subintervals. What can you say about these numbers as the number of subintervals increases?

3. The following sequence of Maple commands reinforces the Fundamental Theorem of Calculus for the definite integral $\displaystyle\int_a^b \frac{1}{1+x^2}\,dx$ with general limits of integration:

```
> f := 1/(1+x^2);                          # enter the integrand
> F := int(f, x);                          # antiderivative of f (F(x))
> eval(F, x=b) - eval(F, x=a);             # area by Fund Thm of Calculus
> int(f, x=a..b);                          # definite integral
```

4. (a) Repeat *Activities* 2&3 for the following definite integrals:

$$\int_0^{\pi/2} \cos(x)\,dx \qquad \int_2^6 x^3\,dx \qquad \int_{-1}^3 e^{-x}\,dx \qquad \int_0^4 \frac{x}{x+1}\,dx \qquad \int_0^4 \frac{x}{x^3+1}\,dx$$

$$\int_0^{3\pi/2} \cos(x)\,dx \qquad \int_0^5 \sqrt{x}\,dx \qquad \int_{-1}^3 xe^{-x}\,dx \qquad \int_0^4 \frac{x}{x^2+1}\,dx \qquad \int_0^4 \frac{x}{x^4+1}\,dx$$

Assignment

Mastery Quiz 11 asks you to use the *Riemann Sums* tutor to approximate and then evaluate a definite integral. The activities in this lab will help you understand the quiz.