

## Critical Points, `fsolve`, and Custom Functions

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### Overview

The analysis of a function via calculus involves solving a variety of equations:  $f'(x) = 0$  for critical numbers,  $f''(x) = 0$  for possible inflection points. These equations are generally easy to formulate but difficult to solve. In fact, in many cases it is impossible to find exact solutions. Maple's `fsolve` command will be used to find approximate solutions to equations.

### Maple Essentials

- The *Graph\_f* and *Graph\_df* maplets provide practice using different types of information about a function to make conclusions about properties of the function. These maplets are available from the course website:

<http://www.math.sc.edu/calclab/141L-S06/labs/> → [Graph\\_f](#)

<http://www.math.sc.edu/calclab/141L-S06/labs/> → [Graph\\_df](#)

- The *Curve Analysis* tutor in Maple 9.5 can be found under the **Tools** menu:

**Tools** → **Tutors** → **Calculus – Single Variable** → **Curve Analysis ...**

- Important Maple commands introduced in this lab include:

Command	Description
<code>fsolve</code>	Similar to <code>solve</code> . Returns a floating-point approximation. (Returns a decimal instead of an exact value.)

### Preparation

Review the First and Second Derivative Tests.

### Assignment

- This week's mastery quiz will be similar to the activities in today's lab.
- Your assignment for this week is to complete **Project 2**. You should prepare a neat and complete project report. All projects are due at the *beginning* of next week's lab.
- Next week's lab will be **Hour Quiz 2**. You should study all material since the last hour quiz.

### Activities

The activities on the back of this lab will help you explore the relationships among the function, the first derivative, and the second derivative. You should practice with the maplets until you are proficient at answering the questions they pose. You should also pay careful attention to the steps for creating a custom function as you will have to create three custom functions in this week's project.

### Problems

1. Use the *Graph\_f* maplet to practice your skills answering questions about the first and second derivative when given the graph of  $f(x)$ .
2. Use the *Graph\_df* maplet to practice your skills answering questions about the original function and second derivative when given the graph of  $f'(x)$ .
3. Use `fsolve` to estimate all critical numbers of  $f(x) = x^{\sin x}$  on  $(0,10)$ .
  - Assign the function to the variable  $f$ , and its derivative to  $df$  using `:=`.
  - Plot the graph of the derivative on the interval  $(0,10)$  to find the general location of the roots.
  - Use `fsolve` with bounds on  $x$  to find the roots one at a time. For example, we can see that the first root lies between 0 and 1, so we would input:  
`> fsolve(df=0, x=0..1);`  
 Find the other three critical points in the same manner.
4. Find and plot the cubic polynomial that has a relative maximum at  $(-1,2)$  and a relative minimum at  $(3,-2)$ .
  - We will want to plot the cubic and the points to ensure that our relative extrema are in the correct locations. We will need to load the `plots` package.  
`> with(plots):`
  - We begin with a generic cubic polynomial and its derivative.  
`> F:= a*x^3 + b*x^2 + c*x + d ;`  
`> dF:= diff(F, x);`
  - We get two equations from the points  $(-1,2)$  and  $(3,-2)$ .  
`> eq1:= eval(F, x=-1) = 2;`  
`> eq2:= eval(F, x=3) = -2;`
  - We get two more equations from  $F'(x) = 0$  at relative extrema.  
`> eq3:= eval(dF, x=-1) = 0;`  
`> eq4:= eval(dF, x=3) = 0;`
  - We now solve the system of 4 equations and 4 unknowns. We will use the `fsolve` command to get numeric approximations instead of fractions.  
`> values:= fsolve({eq1,eq2,eq3,eq4}, {a,b,c,d});`
  - We find the cubic by plugging in these values into  $F$ .  
`> F:= eval(F, values);`
  - We then plot the cubic and the points to verify that we have the correct cubic.  
`> P1:= plot(F, x=-5..5, y=-5..5):`  
`> P2:= pointplot([-1,2], symbolsize=10):`  
`> P3:= pointplot([3,-2], symbolsize=10):`  
`> display([P1,P2,P3]);`
  - Notice that the graph has relative extrema at  $(-1,2)$  and  $(3,-2)$  as desired.
5. Using the **Curve Analysis** tutor, find the intervals over which  $f(x) = x^3 - 5x + 1$  is increasing, decreasing, concave up, and concave down.