Project 2: Building a Better Roller Coaster

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The Problem

Suppose you are asked to design the first ascent and drop for a new roller coaster. By studying your favorite coasters, you decide to make the slope of ascent 0.8 and the slope of the drop -1.6. You decide that for your riders to have a smooth ride you must create the ride based on a piecewise-defined function whose first and second derivatives are both continuous. You decide to connect your straight stretchs $y = L_1(x)$ and $y = L_2(x)$ to part of a parabola $f(x) = ax^2 + bx + c$ by means of two cubic functions. To simplify the equations, you again place the transition point P at the origin. You develop the following piecewise-defined function for your coaster.

$L_1(x)$	$-20 \leq x < 0$
$g(x) = kx^3 + lx^2 + mx + n$	$0 \leq x < 20$
$f(x) = ax^2 + bx + c$	$20 \leq x \leq 100$
$h(x) = px^3 + qx^2 + rx + s$	$100 < x \leq 120$
$L_2(x)$	$120 < x \leq 140$

(1) Write a system of 11 equations in 11 unkowns that ensure that the functions and their first two derivatives agree at the transition points.
Note: You <u>must</u> explain the reasoning for your equations and include the equations within your report.

(2) Solve the equations in (1) with Maple to find formulas for $f(x)$, $g(x)$ and $h(x)$.

(3) Find equations for $L_1(x)$ and $L_2(x)$.
Note: You <u>must</u> include equations for $L_1(x)$, $g(x)$, $f(x)$, $h(x)$, and $L_2(x)$ within your report.

(4) Plot the peicewise-defined function.
Note: Make sure that the individual portions of the graph are distinguishable and labelled.

(5) What is the difference in elevation between P (transition point between $L_1(x)$ and $g(x)$) and Q (transition point between $h(x)$ and $L_2(x)$)?