LAB E: LIMITS, INFINITY, AND ASYMPTOTES

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Overview

Asymptotes for functions are sometimes easy to identify from a graph. The actual definitions of asymptotes are in terms of limits:

Asymptote	Equation	Definition
Horizontal	$y = L$	$\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$
Vertical	$x = a$	$\lim_{x \to a^+} f(x) = \pm\infty$ or $\lim_{x \to a^-} f(x) = \pm\infty$

This lab is designed to provide experience finding asymptotes. Some limits will be evaluated by inspection. You will learn several ways to use Maple to help evaluate more difficult limits.

Maple Essentials

- The *Rational Functions* tutor is started from the Maple 9.5 user interface under the **Tools** menu:
  - **Tools → Tutors → Precalculus → Rational Functions** . . .
- The *LimitCheck* maplet is started from the course website:
  - **www.math.sc.edu/∼sanders/141L-S05/labs/ → LimitCheck(USC)**

Preparation

Review the definitions of horizontal and vertical asymptotes. Also review basic facts about rational functions.

Maple Syntax

Your TA will show you how to use the **Expression** and **Symbol** palettes to avoid typing so much, but the commands for limits are described in the following table:

Limit	Maple Command
$\lim_{x \to a} f(x)$	limit(f(x), x=a);
$\lim_{x \to a^-} f(x)$	limit(f(x), x=a, left);
$\lim_{x \to a^+} f(x)$	limit(f(x), x=a, right);

The constant ∞ is represented in Maple as infinity.

Activities

Your task is to identify all horizontal and vertical asymptotes for as many of the functions on the back of this page as possible. For <u>rational functions</u> the *Rational Functions* tutor can be used to obtain a graph of the function and its asymptotes, but you will still need to use the steps to find the exact equations of the asymptotes.

General Directions

(1) Look at the function $f(x)$ and determine which values make the denominator zero. (You can use the command factor(*expression*); for factoring if necessary.) These values will be the a's that we need to check as possible vertical asymptotes.

(2) Define f as your function and a as one of the values to be checked.

(3) Enter the following lines of code:
   (a) limit(f, x=a, left);
   (b) limit(f, x=a, right);
   If either of these returns the value ∞ or $-\infty$ then $x = a$ is the equation of a vertical asymptote of $f(x)$.

(4) Enter the following lines of code:
   (a) limit(f, x=infinity);
   (b) limit(f, x=-infinity);
   If either of these returns a value $L \neq \pm\infty$ then $y = L$ is the equation of a horizontal asymptote of $f(x)$.

Functions

(1) $f(x) = \frac{3x^2+2x-1}{x+2}$ [This is the default function in the *Rational Functions* tutor.]

(2) $f(x) = \frac{5+2x}{1+x}$

(3) $f(x) = \frac{3x^2+1}{x^2+2x-15}$

(4) $f(x) = \frac{2x^2-x-1}{x^3-2x^2-x+2}$

(5) $f(x) = \left(1 + \frac{3}{x}\right)^x$

(6) $f(x) = \frac{\sqrt{x^2+4}-2}{x}$

(7) $f(x) = \frac{t^3+3t^2-12t+4}{t^3-4t}$

(8) $f(x) = \frac{\sqrt{x^2+1}+2x}{x}$

Assignment

Your assignment for this week is to complete this lab if you did not have the opportunity in your lab period. This material will be included on Maple Quiz 1.