Lab K: Implicit Differentiation

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Overview
This lab provides experience working with functions defined implicitly. The first task is to be able to graph an implicitly-defined function. Next, the equation of the tangent line as a point on the graph of an implicitly-defined function is found and added to the graph.

Maple Essentials

- The *ImplicitDifferentiation* maplet is started from the course website: **www.math.sc.edu/~sanders/141L-S05/labs/** → ImplicitDifferentiation**(TAMU)**
- The Maple commands used in this lab are:

Command	Description
display	combine one or more plots in a single plot; part of the **plots** package
implicitdiff	compute derivatives for implicitly-defined functions
implicitplot	create graph of an implicitly-defined function; part of the **plots** package
simplify	simplify an expression
solve	solve one or more equations for specified variables
with	loads the contents of a Maple package

Preparation
Review imlplicitly-defined functions and implicit differentiation. Also, review the methods for finding and plotting tangent lines.

Activities

(1) Log in and start a Maple session.
(2) Type **with(plots):** at the top of your worksheet. This will allow us to plot points, use the display command, and use the commands for implicitly-defined functions.
(3) **Example 1:** Find the equation of the tangent line to the curve $y^3+yx^2+x^2-3y^2=0$ at the point $(-1,1)$. Then graph the curve and the tangent line with a viewing window of (-5,5)X(-2,4).
- First, to find the tangent line, we need a point and a slope. We have the point $(-1,1)$. We will find the slope by evaluating the derivative at this point. We do the following:
  Define eq as our equation:
  > eq:= y^3 + y*x^2 + x^2 - 3*y^2 =0;
  Define DyDx as the derivative of our equation:
  > DyDx:= implicitdiff(eq, y, x);
  Define m as the derivative evaluated at $(-1,1)$:

> m:= eval(DyDx, {x=-1,y=1});
Define L as the tangent line. Remember $y = m(x - x_1) + y_1$.
> L:= m * (x+1) + 1;

- Next, we write commands that will plot the curve, the point, and the tangent line.
  > P1:= implicitplot(eq, x=-5..5, y=-2..4):
  > P2:= pointplot([-1,1], color=green, symbolsize=15):
  > P3:= plot(L, x=-5..5, color=blue):
- Next, we use the **display** command to view these plots on a single plot.
  > display([P1, P2, P3]);

(4) **Example 2:** Find the equation of the tangent line to the curve $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ at the point $(3, 1)$. Then graph the curve and the tangent line with a viewing window of (-5,5)X(-2,4).

- Follow the steps of Example 1 to solve this problem.

(5) **Example 3:** Find all points where the tangent line to the graph of $x^2y - xy^2 = 2$ is horizontal or vertical.

- We first define eq as our equation.
  > eq:= x^2*y - x*y^2 = 2;
- We then define DyDx as the derivative.
  > DyDx:= implicitdiff(eq, y, x);
- We then simplify so we can identify zeros and undefined x-values.
  > simplify(DyDx);
- The tangent line is horizontal where DyDx=0 and vertical where DyDx=undefined.

Assignment

Your assignment for this week is to complete this lab if you did not have the opportunity in your lab period. This material will be included on Maple Quiz 3.