# Lab F: A More Rigorous Approach to Limits

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**Overview**
The rigorous $\epsilon$-$\delta$ definition of limits can be difficult for students to grasp. This lab is designed to provide visual and interactive tools for working with these concepts.

**Maple Essentials**

- The *EpsilonDelta* maplet is started from the course website:
  - **www.math.sc.edu/∼sanders/141L-S05/labs/** → EpsilonDelta**(USC)**

**Preparation**
We begin with the textbook definition of limit.
Limit Definition. Let $f(x)$ be defined for all $x$ in some interval containing the number $a$, with the possible exception that $f(x)$ need not be defined at $a$. We will write

$$\lim_{x \to a} f(x) = L$$

if given any number $\epsilon > 0$ we can find a number $\delta > 0$ such that

$$|f(x) - L| < \epsilon \text{ if } 0 < |x - a| < \delta.$$

**Maple Syntax**
For precise solutions to our inequalities, we will be using Maple's **solve** command. The general syntax is

solve(eqn, var);

where *eqn* is the equation (or inequality) and *var* is the variable for which we want to solve. We will input most of our inequalities as follows

solve(abs(f(x)−L) < $\epsilon$, x);

For example, if we want to know where $|\sqrt{x} - 2| < 0.05$ we would use the following command

solve(abs(sqrt(x)−2) < 0.05, x);

and Maple would return the interval $(3.8025, 4.2025)$.

**Activities**
When using the $\epsilon$-$\delta$ definition of the limit, we want to find the largest $\delta$ that works in the definition. For each of the limits on the back, your task will be to discover the $\delta$ for each $\epsilon$ given.

**General Directions**

(1) Look at the limit and identify $f(x)$, $L$, $a$, and $\epsilon$.
(2) Launch the *EpsilonDelta* maplet.
(3) Enter the function $f(x)$, $a$, and $L$. Enter an appropriate viewing window. Enter $\epsilon$.
(4) Click plot. You should see the graph of $f(x)$ in blue with brown shading that goes from $x-\delta$ to $x+\delta$ along the $x$-axis and from $f(x-\delta)$ to $f(x+\delta)$ along the $y$-axis. You will notice two red horizontal lines, one at $L-\epsilon$ and the other at $L+\epsilon$.
(5) Your task is to increase the value of $\delta$ as far as possible without shading beyond the red lines. You should zoom several times to insure that you have not crossed either horizontal line.
(6) When you think you are done, write down your last value of $\delta$ that did not cross the line.
(7) Now we will find the value of $\delta$ more precisely.
(8) Use the **solve** command as follows

$$\text{solve(abs(f(x)}-\text{L)} < \epsilon\text{, x);}$$

where $f(x)$, $L$ and $\epsilon$ are all replaced with your values. Maple will return an interval.
(9) Find the distances from $a$ to the left bound and from $a$ to the right bound of the interval. The *smallest* of these two values is the *largest* $\delta$ that works for this $\epsilon$.
(10) Your values from the *EpsilonDelta* maplet and from using the **solve** command should be very close.

**Functions**

(1) $\lim_{x\to 9}\sqrt{x} = 3,\ \epsilon = 0.15,\ \epsilon = 0.05$

(2) $\lim_{x\to 3}(4x-5) = 7,\ \epsilon = 0.4,\ \epsilon = 0.2$

(3) $\lim_{x\to 3}(5x-2) = 13,\ \epsilon = .05,\ \epsilon = .01$

(4) $\lim_{x\to 2}(x^2+3x-1) = 9,\ \epsilon = 0.8,\ \epsilon = 0.6$
Hint: For this one, you should use the interval that contains $a$.

**Assignment**

Your assignment for this week is **Maple Quiz 1**. Be sure to obtain a copy of your quiz before leaving the lab. The quiz will be due at the *beginning* of your lab period next week. You should also complete this lab if you did not have the opportunity in your lab period.