Lab G: Differentiation and Tangent Lines

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Overview
This week's lab will provide practice finding the equation of the tangent line to a curve.

Maple Essentials

- The *Tangents* tutor is started from the Maple 9.5 interface under the tools menu:
  - **Tools → Tutors → Calculus - Single Variable → Tangents** ...
- The *TangentLine* maplet is started from the course website:
  - **www.math.sc.edu/∼sanders/141L-S05/labs/** → TangentLine**(TAMU)**

Preparation
Recall the point-slope form of the equation of the line:

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) is a point on the line and m is the slope of the line. Next, solve the equation for y and we get:

$$y = m(x - x_1) + y_1.$$

Now, we use the substitution $y_1 = f(x_1)$ and this becomes:

$$y = m(x - x_1) + f(x_1).$$

Finally, we know that the derivative evaluated at x_1 is the same as the slope of the tangent line at x_1. Thus we get the following formula for the equation of the tangent line at x_1:

$$y = f'(x_1)(x - x_1) + f(x_1).$$

Maple Syntax
The Maple commands involved with finding and plotting the tangent line to the graph of a (differentiable) function are:

Command	Description
:=	assignment
diff	differentiate an expression
eval	evaluate at a point
plot	plot one or more expressions

Activities
We will find the equation of the tangent line to the graph of $f(x)$ at the point $(x_1, f(x_1))$ for several different functions. We will then graph the function and its tangent line on the same axes.

Example Problem

(1) We will solve the following problem together in two different ways:
- Find an equation for the line that is tangent to the graph of the differentiable function $f(x) = x^3 - 2x + 1$ at $x_1 = 2$. Then graph the curve and this tangent line on the same axes.

(2) The first way:
   (a) Launch the *Tangents* tutor.
   (b) Enter the function as x^3-2*x+1 and the base point as 2.
   (c) Press the **Display** button.
   (d) The maplet will return the graph of the function and the tangent line when you press **Close**.

(3) The second way:
   (a) Enter and execute one by one the following Maple commands.

```
> f := x^3-2*x+1;                    Assign the function to f.
> Df := diff( f,x );                 Compute the derivative f'(x) and assign to Df.
> m := eval( Df,x=2 );               Find f'(2) and assign to m.
> L := m*( x-2 )+eval( f,x=2 );      Find the tangent line y=f'(2)(x-2)+f(2).
> plot( [f, L], x=-2..3 );           Plot the function and the tangent line.
```

Functions

(1) $f(x) = \sqrt{x}, x_1 = \frac{1}{4}$

(2) $f(x) = \frac{5}{x} + 1, x_1 = -2$

(3) $f(x) = x^2, x_1 = 1$

(4) $f(x) = 2^x, x_1 = 1$

(5) $f(x) = \sin(x), x = \frac{\pi}{4}$

Assignment

Your assignment for this week is to complete this lab if you did not have the opportunity in your lab period. This material will be included on Maple Quiz 2.