

# Differentiation and Tangent Lines

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## Overview

We will learn in this lab how to use Maple to find derivatives and the equation of the tangent line to a curve at a given point.

## Maple Essentials

- Important Maple commands introduced in this lab are:

Command	Description	Example
<code>diff(f(x),x);</code>	find derivative of $f(x)$	<code>diff(k^3*x^2,x);diff(k^3*x^2,k);</code>
<code>diff(f(x),x\$n);</code>	find nth derivative of $f(x)$	<code>diff(x^8,x\$4);diff(x*cos(x),x\$100);</code>

- The *Tangents* tutor is started from Maple interface under the tools menu:
  - **Tools** → **Tutors** → **Calculus - Single Variable** → **Tangents ...**
- The *TangentLine* maplet is started from the course website:
  - <http://people.math.sc.edu/calclab/141L-F18/labs/> → TangentLine

## Related course material/Preparation

§3.1 and §3.2. Recall the point-slope form of the equation of the line:

$$y - y_1 = m(x - x_1),$$

where  $(x_1, y_1)$  is a point on the line and  $m$  is the slope of the line. Next, since point  $(x_1, f(x_1))$  is on the tangent line, we can substitute  $y_1 = f(x_1)$  and move it to the other side. We hence get:

$$y = m(x - x_1) + f(x_1).$$

Finally, we know that the derivative evaluated at  $x_1$  is the same as the slope of the tangent line to the graph of  $y = f(x)$  at  $x_1$ . Thus we get the following formula for the equation of the tangent line to the graph of  $y = f(x)$  at  $x_1$ :

$$y = f'(x_1)(x - x_1) + f(x_1).$$

## Activities

A) We already knew how to find the derivative from the definition, that is, use Maple to find the limit

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

A more direct way is to use the command `diff`. To compute the derivative of  $f(x)$  with respect to  $x$ , you simply type `diff(f(x),x)`; (or use the right-clicking). It can also be used to find higher order derivatives. For example, to find the third derivative of  $x^4$ , you simply type `diff(x^4,x$3)`; . Try the following sets of examples (please pay attention to subtle differences):

1. `diff(k*x^4,x); diff(k*x^4,k); diff(k*x^4,x$4); diff(k*x^4,x$5);`
2. `f:=x->x^4; diff(f,x); diff(f(x),x); diff(f(sin(x)),x);`
3. `diff(sin(x)*cos(x),x$100); diff(x*x*cos(x),x$8);`

**B)** Find the equation of the tangent line to the graph of  $f(x)$  at the point  $(x_1, f(x_1))$  for the following functions. Graph the function and its tangent line on the same axes.

1.  $f(x) = x^2, x_1 = 1$
2.  $f(x) = 2^x, x_1 = 1$
3.  $f(x) = \cos(x), x_1 = \frac{\pi}{4}$  (Recall that, in Maple, you type Pi for  $\pi$ )

### Example Problem

We will do an example together for  $f(x) = x^3 - 2x + 1$  at  $x_1 = 2$  in two different ways:

The first way:

1. Launch the *Tangents* tutor.
2. Enter the function as  $x^3-2*x+1$  and  $x=2$ , and change the number of iterations to 5.
3. Click **Display**. The tutor will display the function and a series of secant lines, including the tangent line. The equation of the tangent line is displayed on the right.
4. Press the **Animate** button. The tutor will show the progression through the secant lines as  $\Delta x$  gets smaller.
5. The tutor will return the last graph when you click **Close**.
6. If you want to graph the function and the tangent line, assign both in a Maple worksheet and write a plot command.

The second way:

1. Define the function and assign it to  $f$ .  
 $> f := x \rightarrow x^3-2*x+1;$
2. Right-click and choose *Differentiate*. Then use a label (or right-click again) to assign this new function to  $df$ .  
 $> df := label;$
3. Find  $f'(2)$  and assign that value to  $m$ .  
 $> m := df(2);$
4. Find the equation of the tangent line  $y = f'(2)(x - 2) + f(2)$  and assign it as a function to  $L$ .  
 $> L := x \rightarrow m*(x-2)+f(2);$
5. Plot the function and the tangent line using different linestyles.  
 $> plot([f(x), L(x)], x=-1..3, linestyle=[solid, dash]);$

**C)** The *TangentLine* maplet is a great tool to practice finding the equation of the tangent line by hand. Launch the maplet and click **New Function**. Follow the prompts step by step to find  $f(a)$ ,  $f'(x)$ , and  $f'(a)$ . Then enter the equation of the tangent line as follows:

$$y = f'(a)(x - a) + f(a).$$

The maplet will check each of your answers (or on Show if you don't know the answer) and let you know whether you are correct. Please try a few problems to make sure that you really understand how to find the equation of the tangent line by hand.

### Assignment

Complete lab activities and your lab instructor will give other assignment for each section.