New Functions from Old

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Overview

In this lab, we will use Maple to help us calculate and simplify combinations of functions. The **Shift** maplet will also be introduced to help us practice our skills identifying basic functions that have been shifted horizontally and/or vertically.

Maple Essentials

- The *Shift* maplet is available from the course website:

   <div align="center">

   `http://www.math.sc.edu/calclab/141L-F08/labs/` → <u>Shift</u>

   </div>

- New Maple commands introduced in this lab include:

Command	Description
`simplify`	`simplify(f(x));` simplifies $f(x)$

Preparation

Read Section 1.3: *New Functions from Old* in Anton.

Activities

1. Use the **Shift** maplet to practice your skills identifying basic functions that have been shifted horizontally and/or vertically.

   (a) From the Calculus I Lab Assignments page under Lab C, click on <u>Shift</u>. You will be prompted for a username and password as these maplets are protected. You should use your Blackboard username and password.

   (b) This opens a user interface for testing your ability to recognize shifts of seven basic functions. To see the seven basic functions, click the **Show Basic 7 Functions** button.

   (c) To test your ability to recognize shifts of these functions, click on the **Show Shifted Graph** button. Enter the formula for the displayed graph (using valid Maple syntax) in the box labeled *Answer*, then click the **Check Answer** button.

   Note: *If you do not get the answer correct, the graph of your equation will be displayed in red.*

2. In each of the problems on the next page, you will use the assignment operator (`:=`) together with the arrow notation (`->`) to define each function. Once you have done this, the problems are straightforward.
   Note: *Remember that you can use the Expression and Common Symbols palettes to avoid typing so much. You may also find the labels useful.*

- Find and simplify formulas for $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $(f/g)(x)$.
  a. $f(x) = 2\sqrt{x - 1}$, $g(x) = \sqrt{x - 1}$ (Ex. 29 on Page 37)
  b. $f(x) = 1 + \frac{x}{x+1}$, $g(x) = 2 - \frac{1}{x}$
- Let $f(x) = x^2 + 1$. Find and simplify each of the following. (Ex. 34 on Page 37)
  a. $f(t + 2)$ b. $f(\sqrt{x})$ c. $f(f(x))$
- Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$. Simplify your answer.
  a. $f(x) = 3x^2 - 5$ (Ex. 53 on Page 38)   b. $f(x) = \frac{1}{(x+1)^2}$
- Find and simplify $(f \circ g)(x)$ and $(g \circ f)(x)$.
  a. $f(x) = \frac{1+x}{1-x}$, $g(x) = \frac{x}{1-x}$ (Ex. 37 on Page 37)
  b. $f(x) = \sqrt{2x + 3}$, $g(x) = x^2 + 1$
- Find and simplify $(f \circ g \circ h)(x)$.
  a. $f(x) = \sqrt{x - 1}$, $g(x) = x^2 + 2$, $h(x) = \cos x$
  b. $f(x) = x^2 + 1$, $g(x) = \frac{1}{x}$, $h(x) = x^3$ (Ex. 39 on Page 37)
- Express F as a composition of two functions; that is, find f and g such that $F = (f \circ g)(x)$. Use Maple to verify the composition. (Ex. 43 on Page 37)
  Note: *Do not choose the identity $(y = x)$ as one of your functions.*
  a. $F(x) = \sin^2 x$ b. $F(x) = \frac{3}{5+\cos x}$ c. $F(x) = (x^2 + 1)^{10}$
- Express F as a composition of three functions; that is, find f, g, and h such that $F = (f \circ g \circ h)(x)$. Use Maple to verify the composition. (Ex. 45 on Page 37)
  Note: *Do not choose the identity $(y = x)$ as one of your functions.*
  a. $F(x) = (1 + \sin(x^2))^3$ b. $F(x) = \sqrt{1 - x^{1/3}}$ c. $F(x) = \cos^4(\sqrt{x})$

Example Problems

1. Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$ if $f(x) = \frac{4}{3+x^2}$. Simplify your answer.
   ```
   > f:= x -> 4 / (3+x^2);
   > (f(x+h) - f(x)) / h;
   > simplify(label);
   ```
   Note: *You should right-click over your expression and choose an action. If you choose to type the command, use* **Ctrl-L** *to insert a label.*

2. Find and simplify $(f \circ g \circ h)(x)$ if $f(x) = \frac{2}{1-x^2}$, $g(x) = \sin(x)$, and $h(x) = \sqrt{x}$.
   ```
   > f:= x -> 2 / (1-x^2);
   > g:= x -> sin(x);
   > h:= x -> sqrt(x);
   > f(g(h(x)));
   > simplify(label);
   ```

Assignment

With the help of Maple, work out exercises 12, 30, 32, and 56 of §1.3 (pages 36-38). Write your answers on a sheet of paper to be turned in with your Maple worksheet.