Implicit Differentiation

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Overview

This lab provides experience working with functions defined implicitly.

Maple Essentials

- Important Maple commands introduced in this lab are:

Command	Description	Example
`display`	display plots in a single plot (need `plots` package)	`display([F,G],title=‘‘Fig1’’);`
`implicitplot`	create graph of function defined implicitly (need `plots` package)	`implicitplot(x*y=1,x=0..1,y=0..1);`
`pointplot`	plot points (need `plots` package)	`pointplot([1,2], color=red, symbolsize=18):`
`implicitdiff`	compute derivatives of functions defined implicitly	`implicitdiff(f,y,x);` `implicitdiff(f,y,x$2);`
`fsolve`	compute a solution of equations numerically	`fsolve({f=1,g=x^2},{x,y});` `fsolve({f,g},{x,y},{x=0..1,y=0..2});`
`with`	load a Maple package	`with(plots):  with(plots);`

- The *ImplicitDifferentiation* maplet is available from the course website:
  http://www.math.sc.edu/calclab/141L-F07/labs → ImplicitDifferentiation

Related course material/Preparation

§4.1 Implicit Differentiation (Pages 235-247) of the textbook.

Assignment

Exercises 19, 21, 26, 39, and 40 (pages 241-242).

Hint for 39 and 40: For part a), start with a big range for both x and y in `implicitplot` to see the size of the view window the graph will display and then re-plot the graph with that view window for a better plot. For part c), you also need to specify different regions in `fsolve` according to your graph to get all solutions, as `fsolve` only computes one solution at a time.

Activities

Problem 1: Find the equation of the tangent line to the curve $2(x^2+y^2)^2 = 25(x^2-y^2)$ at the point $(3,1)$. Then graph the curve, the point, and the tangent line with a viewing window of (-5,5)x(-2,4).

Problem 2: Find all points where the tangent line to the graph of $x^2y - xy^2 = 2$ is horizontal or vertical. (Hint: The tangent line is vertical where $dx/dy = 0$.)

Problem 3: Find d^2y/d^2x and d^3y/d^3x if y is defined implicitly by $y + \sin y = x$ (Ex.25 on page 242).

Example Problem

We will redo example 5 on page 239 together using Maple:

- a) Use implicit differentiation to find dy/dx for the Folium of Descartes $x^3 + y^3 = 3xy$.
- b) Find the equation of the tangent line to the Folium of Descartes at the point $(3/2, 3/2)$. (Then graph the curve, the point, and the tangent line with a viewing window of (-3,3)x(-4,3) as shown in Figure 4.1.5 on page 239.)
- c) At what point(s) in the first quadrant is the tangent line to the Folium of Descartes horizontal?

Steps:

1. Start a Maple session with `restart;` and load the Maple `plots` package. This package allows us to plot points, use the display command, use the commands for implicitly-defined functions, and more. Notice that we used '`:`' instead of '`;`'. The difference is that the maple does not display the output with '`:`'.
   ```
   > restart;
   > with(plots):
   ```
2. For part a), simply assign the Folium of Descartes to, say, `FD`, then use command `implicitdiff` to find dy/dx.
   ```
   > FD:=x^3 +y^3 =3*x*y;
   > dydx:=implicitdiff(FD,y,x);
   ```
   (Notice that `implicitdiff(f,x,y);` computes dx/dy and `implicitdiff(f,y,x$n);` computes $d^n y/d^n x$. You will need them to do problem 2 and problem 3, respectively.)
3. Next, to find the tangent line, we need a point and a slope. The point $(3/2, 3/2)$ is given and we find the slope `m` by evaluating dy/dx at this point.
   ```
   > m:= eval(dydx, {x=3/2, y=3/2});
   ```
4. Find the equation of the tangent line by the point-slope formula $y = m(x - x_1) + y_1$.
   ```
   > L:=x-> m*(x-3/2)+3/2;
   ```
5. Next, write (and assign) commands to plot the curve, the point, and the tangent line. Write the commands separately using ':' so Maple does not display the output yet. (In the first plot command, the option `numpoints=10000` will insure a smooth curve.)
   ```
   > P1:= implicitplot(FD, x=-3..3, y=-4..3, numpoints=10000):
   > P2:= pointplot([3/2,3/2], color=green, symbolsize=15):
   > P3:= plot(L(x), x=-3..3, y=-4..3, color=blue, linestyle=DOT):
   ```
6. These plots can then be displayed on a single plot using the `display` command.
   ```
   > display([P1, P2, P3], title=``Figure 1'');
   ```
7. From the graph, we can see that the answer to part c) is a point located approximately at $(1.2, 1.5)$. Since this point is on the curve and the $dy/dx = 0$ at this point, we can find it's location by solving those two equations.
   ```
   > fsolve({FD,dydx=0},{x,y},{x=1..2,y=1..2});
   ```
   (For a numerical solution in a specified region, `fsolve` in general does a better job than `solve`.)

Additional Notes

The ImplicitDifferentiation maplet provides additional practice finding the slope of a curve at a point.