Overview
The rigorous $\epsilon$–$\delta$ definition of limits can be difficult for students to grasp. This lab is designed to provide visual and interactive tools for working with these concepts.

Maple Essentials
- The EpsilonDelta maplet is available from the course website:
  \[ \text{http://www.math.sc.edu/calclab/141L-F07/labs/} \rightarrow \text{EpsilonDelta} \]

Related course material/Preparation
$\S$2.4 (Pages 134-143) of the textbook (Anton, 8th edition). Let us first recall the definition of limit given there: Let $f(x)$ be defined for all $x$ in some open interval containing the number $a$, with the possible exception that $f(x)$ need not be defined at $a$. We will write $\lim_{x \to a} f(x) = L$ if given any number $\epsilon > 0$ we can find a number $\delta > 0$ such that

$$|f(x) - L| < \epsilon \text{ if } 0 < |x - a| < \delta.$$  

In general, $\epsilon$ and $\delta$ are meant to be very small numbers. Therefore, intuitively, the definition states that $f(x)$ will be very close to $L$ that is, $|f(x) - L| < \epsilon$, when $x$ is very close to $a$ ($|x - a| < \delta$). The task is to show that, for any given $\epsilon$ (no matter how close $f(x)$ is to $L$), you can always find a $\delta$–needed closeness of $x$ to $a$–to make it work.

Activities
From our discussion, our job is to find a $\delta$ for a given $\epsilon$ such that, when $a - \delta < x < a + \delta$, the inequality $|f(x) - L| < \epsilon$ holds. Therefore, we need to solve for a range $a - \delta < x < a + \delta$ of $x$ from the given inequality $|f(x) - L| < \epsilon$. Ideally, we would like to find a formula of $\delta$ in terms of $\epsilon$ (see examples 1, 2, and 3 of $\S$2.4) that will work for any given $\epsilon$. However, such formulas are in general very hard to find. Moreover, the value of $\delta$ is not unique, as any value that is smaller than a solution would work, too. For each of the limits below, we will use Maple’s solve command to help us to find the largest $\delta$ that works for the given $\epsilon$ and the interactive EpsilonDelta maplet provides a tool to visualize relations between $\delta$ and $\epsilon$.

(Follow the General Directions on the back of this page.)

1. $\lim_{x \to 9} \sqrt{x} = 3$, $\epsilon = 0.15$, $\epsilon = 0.05$

2. $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = 6$, $\epsilon = 0.2$, $\epsilon = 0.05$

3. $\lim_{x \to 3} (5x - 2) = 13$, $\epsilon = 0.05$, $\epsilon = 0.01$

4. $\lim_{x \to 2} (x^2 + 3x - 1) = 9$, $\epsilon = 0.8$, $\epsilon = 0.6$

   HINT: Since we also have $\lim_{x \to 5} (x^2 + 3x - 1) = 9$, Maple’s solve command will return extra solutions. Which interval should you choose for problem 4?
**Use Maple’s solve command to solve inequalities**

Maple’s `solve` command was introduced in the Lab 4 to solve equations. It can also be used to solve inequalities. We will input most of our inequalities as follows:

```maple
> solve(abs(f(x) - L) < ϵ, x);
```

For example, if we want to know where \( |\sqrt{x} - 2| < 0.05 \) we would use the following command

```maple
> solve(abs(sqrt(x) - 2) < 0.05, x);
```

and Maple would return the interval \((3.8025, 4.2025)\) as the solution (your TA will explain Maple’s notation)

**General Directions**

1. Look at the limit and identify \( f(x), L, a, \) and \( ϵ \).
2. Launch the `EpsilonDelta` maplet and click **Modify or Make Your Own Problem**. Enter the function \( f(x), a, L, \) and \( ϵ \).
3. Click **Save Problem and Close**. You should see the graph of \( f(x) \) in blue with a cyan vertical stripe that goes from \( a - δ \) to \( a + δ \) and a pink horizontal stripe that goes from \( L - ϵ \) to \( L + ϵ \). You should also see a brown rectangle extends vertically from the smallest value of \( f(x) \) to the largest value of \( f(x) \) for \( x \) from \( a - δ \) to \( a + δ \). You may change the size of this rectangle by changing the value of \( δ \), which can be done using the slider (for \( 0.1 ≤ δ ≤ 1 \)) or by typing in (any value).
4. Your task is to determine the largest value of \( δ \) that keeps the brown rectangle completely inside the pink stripe. You can use **Zoom In** to increase the accuracy.
5. When you think you are done, record your final value of \( δ \).
6. Now we will find the value of \( δ \) more precisely using Maple’s `solve` command.
7. Use the arrow notation (`:=x->`) to define the function \( f(x) \). Use `:=` to assign \( L, a, \) and `epsilon` to their respective values.
8. Use the `solve` command as follows

```maple
> solve(abs(f(x) - L) < epsilon, x);
```

Maple should return an interval or intervals.
9. Choose the interval that contains \( a \). Find the distances from \( a \) to the left bound and from \( a \) to the right bound of the interval (both of them should be positive.) The *smallest* of these two values is the *largest* \( δ \) that works for this \( ϵ \).
10. Your values from the `EpsilonDelta` maplet and from using the `solve` command should be very close.

**Remark:**

For some simple functions like linear functions, `solve` can be used to find general formulas of \( δ \) in term of \( ϵ \). Try the following and compare it to problem 3:

```maple
solve(abs(5*x-2-13) < epsilon, x) assuming epsilon>0;
```

**Assignment**

Exercises 9, 12, and 13 in §2.4 on page 141. Review Labs 1-5 for next week’s Hour Quiz 1 (to be completed in lab).