Designer Functions/Project 2

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Overview

The analysis of a function via calculus involves finding its asymptotes (both horizontal and vertical), critical points, intervals of increase and decrease, local extrema, inflection points, and concavity using limits and derivatives. This often leads to variety of equations. These equations are generally easy to formulate but difficult to solve. In fact, in many cases it is impossible to find exact solutions. Maple can be used to help setting up those equations and finding solutions (approximately, if needed).

Maple Essentials

- The *Graph_f* and *Graph_df* maplets provide practice using different types of information about a function to make conclusions about properties of the function. These maplets are available from the course website:

  `http://www.math.sc.edu/calclab/141L-F07/labs/` → Lab 12, last column

Related course material/Preparation

§5.1, §5.2, and §5.3 of the textbook.

Activities

The activities of this lab will help you explore the relationships among the function, the first derivative, and the second derivative. You should pay careful attention to the steps for creating a custom function as you will have to create three custom functions in project 2.

Problems

1. Follow your TA and use the *Graph_f* and *Graph_df* maplets to practice your skills answering questions about the first and second derivative when given the graph of $f(x)$ and questions about the original function and second derivative when given the graph of $f'(x)$, respectively.
2. Find and plot the cubic polynomial that has a relative maximum at (-1,2) and a relative minimum at (3,-2).
   - We will need to load the `plots` package as we want to plot the cubic and the points to ensure that our relative extrema are in the correct locations.
     `> with(plots):`
   - We begin with a generic cubic polynomial and its derivative.
     `> F:= x -> a*x^3 + b*x^2 + c*x + d ;`
     Get the derivative by right clicking and assign it to dF.
   - We get two equations from the points (-1,2) and (3,-2).
     `> eq1:= F(-1) = 2;`
     `> eq2:= F(3) = -2;`
   - We get two more equations from $F'(x) = 0$ at relative extrema.
     `> eq3:= dF(-1) = 0;`
     `> eq4:= dF(3) = 0;`

- We now solve the system of 4 equations and 4 unknowns. (If we used the **fsolve** command, we would get numeric approximations instead of fractions.)
  `> values:= solve({eq1,eq2,eq3,eq4}, {a,b,c,d});`
- We find the cubic by plugging in these values into F.
  `> assign(values);`
  `> F(x);`
- We then plot the cubic and the points to verify that the cubic has has relative extrema at (-1,2) and (3,-2) as desired.
  > P1:= plot(F(x), x=-10..10, y=-10..10):
  > P2:= pointplot([-1,2], symbolsize=10):
  > P3:= pointplot([3,-2], symbolsize=10):
  > display([P1,P2,P3], title="My Graph");

Assignment/Project 2

- Your assignment is **Project 2** creating three designer functions. You should prepare a neat and complete project report containing the following information for each of the three functions:
  - An explanation of how the stated conditions lead to a system of equations for the unknown parameters in the function.
  - The solution to the system of equations and the function that meets all of the conditions.
  - A plot of the function on an appropriate interval that shows all of the essential qualitative features of the function.

  In addition, for the quintic polynomial found in 2, find all zeros of the function and the intervals where the function is increasing, decreasing, concave up, and concave down. Be creative in your presentation of these results. If, for example, you use a table, remember that the main text needs to explain what information is contained in the table. All floating point numbers should be reported with an accuracy of at least three digits to the right of the decimal point. The due date will be specified by your TA.

  These are the functions you need to create:

  1. Find the cubic polynomial $F(x) = ax^3 + bx^2 + cx + d$ with a relative minimum at (*,*) and a relative maximum at (*,*) to be specified by your TA.
  2. Find the quintic polynomial $F(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ that passes through the point (-3,2), has a tangent line at $x = 0$ with slope -1, an inflection point at (2,1) and a local **maximum** at (6,-8). **Hint/Remark:** You should find a system of 6 equations.
  3. Find **all** functions of the form $F(x) = \frac{a+be^x}{c+e^x}$ with asymptotes $y = -3$, $y = 2$, and $x = \ln(8)$, where e^x is the natural expontial function. **Hint/Remark:** You do not have to find any derivatives to solve this problem. In fact, it might be easier to solve this problem by hand. Think about how to use vertical and horizontal asymptotes to discover equations. (You may want to review Lab 4.) If you don't want to see the discontinuity in your graph, you should use *discont=true* in your plot command.