Limits, Infinity, and Asymptotes

Douglas Meade, Ronda Sanders, and Xian Wu
Department of Mathematics

Overview

Asymptotes for functions are sometimes easy to identify from a graph. The actual definitions of asymptotes are given in terms of limits. There are many different types of asymptotes and the two simplest ones are:

Asymptote	Equation	Definition
Horizontal	$y = L$	$\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$
Vertical	$x = a$	$\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$

This lab is designed to provide experience finding those two types of asymptotes. We will also learn several ways to use Maple to help evaluate limits.

Maple Essentials

- Important Maple commands introduced in this lab are:

Command	Description	Example
`limit(f(x),x=a);`	evaluate $\lim_{x\to a} f(x)$	`limit(f(x),x=2);limit(x^2,x=infinity);`
`limit(f(x),x=a,right);`	evaluate $\lim_{x\to a^+} f(x)$	`limit((f(x)+1)/(x^2-1),x=1,right);`
`limit(f(x),x=a,left);`	evaluate $\lim_{x\to a^-} f(x)$	`limit((sqrt(x^2+1)/(x+1),x=-1,left);`
`factor`	factor an expression	factor(f(x)); factor((x^4-1);

  Your TA will show you how to use the **Expression** and **Symbol** palettes to avoid typing so much.

- The *Rational Functions* tutor is started from the Maple 9.5 user interface under the **Tools** menu:
  - **Tools → Tutors → Precalculus → Rational Functions** ...
- The *LimitCheck* maplet is started from the course website:
  - **www.math.sc.edu/calclab/141L-F05/labs/** → LimitCheck**(USC)**

Related course material

§2.1, §2.2, and §2.3 (Pages 101-134) of the textbook.

Activities

A) Your first task is to identify all horizontal and vertical asymptotes for functions 1 to 4 on the back of this page. Since, functions blow up near their vertical asymptotes, you need to specify appropriate ranges for both x and y in order to get nice looking graphs. For rational functions the *Rational Functions* tutor can be used to obtain a graph of the function and its asymptotes, but you will still need to use the following steps to find the exact equations of the asymptotes.

General Directions

1. Look at the function $f(x)$ and determine which values make the denominator zero. (You can use the command factor(*expression*); for factoring if necessary.) These values will be the a's that we need to check as possible vertical asymptotes.
2. Define f as your function and a as one of the values to be checked.
3. Depends on the way that you define your functions, enter either

(a) limit(f, x=a, left); or limit(f(x), x=a, left);
(b) limit(f, x=a, right); or limit(f(x), x=a, right);

If either of these returns the value ∞ or $-\infty$ then $x = a$ is the equation of a vertical asymptote of $f(x)$.

4. Depends on the way that you define your functions, enter either
   (a) limit(f, x=infinity); or limit(f(x), x=infinity);
   (b) limit(f, x=-infinity); or limit(f(x), x=-infinity);

   If either of these returns a value $L \neq \pm\infty$ then $y = L$ is the equation of a horizontal asymptote of $f(x)$.

Functions

1. $f(x) = \frac{3x^2+2x-1}{x+2}$ [This is the default function in the *Rational Functions* tutor.]
2. $f(x) = \frac{5+2x}{1+x}$
3. $f(x) = \frac{3x^2+1}{x^2+2x-15}$
4. $f(x) = \frac{2x^2-x-1}{x^3-2x^2-x+2}$
5. $f(x) = \left(1 + \frac{3}{x}\right)^x$
6. $f(x) = \frac{\sqrt{x^2+4}-2}{x}$
7. $f(x) = \frac{t^3+3t^2-12t+4}{t^3-4t}$
8. $f(x) = \frac{\sqrt{x^2+1}+2x}{x}$

B) Use the Maple to find $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ for the following $f(x)$ (as covered in §3.2, this limit = the derivative of $f(x)$):

1. $f(x) = x^2$
2. $f(x) = 1/x$
3. $f(x) = \sin x$

C) If you have time left, use the Maple to check answers for some home work problems on limits.

Assignment

1) Identify all horizontal and vertical asymptotes for functions 5 to 8 on this page.

2)Answer the following questions:

- What property of a rational function determines whether it has a horizontal asymptote?
- Does every hole in the domain of a function lead to a vertical asymptote?
- Can the graph of a function cross the graph of its horizontal asymptotes? Its vertical asymptotes?
- How many horizontal asymptotes can a graph have?