Function Analysis

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Overview

The objective of this lab is to use Maple to help you practice your function analysis skills. Several user interfaces are introduced as follows.

Maple Essentials

- The *Curve Analysis* tutor is started from the Maple 9.5 user interface under the Tools menu:

  Tools → Tutors → Calculus - Single Variable → Curve Analysis ...

  This tutor calculates extrema and intervals of increasing, decreasing, concave up, and concave down of a user-specified function. Be sure to specify the domain by entering appropriate values in the boxes labeled `a =` *and* `b =`*. The* `Display` *button displays a color charted plot of the function on the specified domain. Click on the* `color` *button to see the meaning of (or to change) the color chart. This is a great tool to use when you need to check your answer to a problem.*

- The *FunctionAnalyzer* maplet is available from the course website:

  http://www.math.sc.edu/calclab/141L-F05/labs → FunctionAnalyzer (USC)

  This maplet is designed to help you to carry out individual steps involved in analysis of a user-specified function. It will find the first and the second derivatives of the given function and generate a plot of your choice. It also has a `Solve an Equation or Inequality` *window, which can be used to find zeros, critical points, inflection points, and more. Similarly, you can use the window* `Evaluate Expression` *to find the value of any function at a given point and to check, for example, signs of the first and the second derivatives. It can even evaluate limits. This is a great tool to use when you need some help to carry out involved calculations.*

- Another set of four similar maplets are available from the course website:

  http://www.math.sc.edu/calclab/141L-F05/labs → GraphTerms, Graph f, Graph df, Graph ddf

  Those maplets ask questions in function analysis with randomly generated graphs of different types. All of them will check user's answers, give hints, or show answers. They are very useful if you need some practice problems with instant feedback.

Related course material

§5.1, §5.2, §5.3, and §5.4 of the textbook. Be sure to recall what covered in those sections.

Activities

1. For each of the functions listed below, do a full analysis (see General Steps on the back for details) over the specified domain with the assistance of the *FunctionAnalyzer* maplet.
   (a) $y = x + 1/x, -5 \le x \le 5$
   (b) $y = x^2 e^{1-x}, -2 \le x \le 10$
   (c) $y = (\sin(x) - \cos(x))^2, -\pi \le x \le \pi$
   (d) $y = (x-2)/(x^2 - x - 1), -8 \le x \le 8$
2. Answer at least one set of questions for each of the *GraphTerms, Graph f, Graph df*, and *Graph ddf* maplets. (Launch the maplet and follow the instructions.)

Full Function Analysis: General Steps

Although Problem 1 (a) can be solved easily without any assistance of a computer, we will work it out together using Maple to review general steps. The idea is to let Maple do all calculations so we can focus on the concept.

1. Launch the *FunctionAnalyzer* tutor. Enter `Function y=` as `x+1/x` and press the `Tab` key. Notice that y' (`Dy`)and y'' (`D2y`) have been computed for you.
2. To find critical points, first `Enter equation to be solved` as `Dy=0` and click on the `Attempt to solve equation` button; this returns ``‘‘Dy = 0, for x={-1, 1}’’``. Next, to find where y' DNE, replace `Dy=0` by `1/Dy=0` and click on the `Attempt to solve equation` button; this returns ``‘‘1/Dy = 0, for x={0}’’``. Therefore, there are three critical points (two stationary points and one singular point) at x=-1, 0, and 1.
3. To check the sign of y' in each interval, enter `Evaluate the expression` as `Dy`, and `x has the value` as, say, `-2`, and click on the `Evaluate expression at specified value` button; this returns ``‘‘Dy = 3/4, when x = -2’’`` (To evaluate 3/4, right click on the window and choose `Manipulate Results` and the `evalf`). Therefore, $y' > 0$ and y is hence increasing on interval $(-\infty, -1)$ (or $[-5, -1)$). Similarly, checking at x=-0.5, 0.5, and 2, we have that y is also increasing on interval $(1, \infty)$ (or $(1, 5]$) and decreasing on intervals $(-1, 0)$ and $(0, 1)$.
4. Similarly, we can find inflection points and the concave structure of y by checking y'' (`D2y`) instead of y' (`Dy`). For the concave structure, don't forget also to check where D2y DNE.
5. The above analysis of y' also tells us y has a relative maximum at x=-1 and a relative minimum at x=1. Evaluating y at x=-1 and x=1, we hence have that the function has a relative maximum of -2 at x=-1 and a relative minimum of 2 at x=1. Notice that a relative maximum can be less than a relative minimum.
6. To find the absolute extrema of y, we need to evaluate y at all critical points and the endpoints, that is, at x=-5, -1, 0, 1, and 5. You will get an `Error` when evaluating at x=0, as our y is not defined at $x = 0$. Recall that, in cases like this or when the interval is open, we need to check limits. To do that, enter `Evaluate the expression` as `Limit(y,x=0,left)` and click on the `Evaluate expression at specified value` button; this returns ``‘‘Limit(y,x = 0,left) = -infinit’’``, that is, $\lim_{x\to 0-} y = -\infty$. Similar, we have $\lim_{x\to 0+} y = \infty$. Therefore, y has neither an absolute maximum nor an absolute minimum on $[-5, 5]$.
7. To verify your analysis geometrically, enter the view-range, say, from -5 to 5 for both x and y for this particular problem. Check the box/boxes to choose the graph/graphs to be included in the plot and click on the `Update plot` button. You can change the color and the style for each of the graphs and check the boxes to display sign charts of y' and y''.
8. Finally, if you like to check your answers numerically, launch the `Curve Analysis` tutor.

Assignment/Quiz 3

The assignment of this week is Quiz 3:

Directions

- The quiz is due before the Thanksgiving break.
- This is an open-book quiz but you **must** complete it individually.
- Present your answers on a separate sheet(or sheets) of paper with your name and student number.
- All values should be correct to 4 places beyond the decimal.
- Direct all questions to your TA.

Questions

1. List all maple commands introduced and give at least one example for each command.
2. Carry out a full function analysis for $y = x^3 - x^{2/3}$ over the interval $[-2, 2]$. You need to show detailed steps to justify your answers but may use maple to help with calculations when necessary.