

# Differentiation and Tangent Lines

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## Overview

We will learn in this lab how to use Maple to find derivatives and the equation of the tangent line to a curve at a given point.

## Maple Essentials

- Important Maple commands introduced in this lab are:

Command	Description	Example
<code>diff(f(x),x);</code>	find derivative of $f(x)$	<code>diff(x^2,x);diff(k*x^2,x);diff(x^3,x\$3)</code>
<code>eval(f,x=p);</code>	evaluate $f(x)$ for $x=p$	<code>eval(x^2,x=2);eval(f,x=5*sin(x))</code>

- The *Tangents* tutor is started from the Maple 9.5 interface under the tools menu:
  - **Tools** → **Tutors** → **Calculus - Single Variable** → **Tangents ...**
- The *TangentLine* maplet is started from the course website:
  - [www.math.sc.edu/calclab/141L-F05/labs/](http://www.math.sc.edu/calclab/141L-F05/labs/) → TangentLine(TAMU)

## Related course material

Recall the point-slope form of the equation of the line:

$$y - y_1 = m(x - x_1),$$

where  $(x_1, y_1)$  is a point on the line and  $m$  is the slope of the line. Next, solve the equation for  $y$  and we get:

$$y = m(x - x_1) + y_1.$$

Now, since point  $(x_1, f(x_1))$  is on the tangent line, we use the substitution  $y_1 = f(x_1)$  and this becomes:

$$y = m(x - x_1) + f(x_1).$$

Finally, we know that the derivative evaluated at  $x_1$  is the same as the slope of the tangent line to the graph of  $y = f(x)$  at  $x_1$ . Thus we get the following formula for the equation of the tangent line to the graph of  $y = f(x)$  at  $x_1$ :

$$y = f'(x_1)(x - x_1) + f(x_1).$$

## Activities

- A) We already knew how to find the derivative from the definition, that is, use Maple to find the limit

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

A more direct way is to use the command `diff`. To compute the derivative of  $f(x)$  with respect to  $x$ , you simply type `diff(f(x),x);`. It can also be used to find higher order derivatives. For example, to find the third derivative of  $x^4$ , you simply type `diff(x^4,x$3);`. Now try the following sets of examples (please pay attention to subtle difference):

1. `diff(k*x^4,x); diff(k*x^4,k); diff(k*x^4,x$4); diff(k*x^4,x$5);`

2. `f:=x^4; g:=x->x^4; diff(f,x); diff(g(x),x); diff(g(sin(x)),x);`
3. `df:=diff(sin(x)*cos(x),x$100); df:=diff(x^x*cos(x),x$100);`

**B)** We will find the equation of the tangent line to the graph of  $f(x)$  at the point  $(x_1, f(x_1))$  for a few functions listed below and then graph the function and its tangent line on the same axes.

### Example Problem

1. We will solve the following problem together in two different ways:
  - Find an equation for the line that is tangent to the graph of the differentiable function  $f(x) = x^3 - 2x + 1$  at  $x_1 = 2$ . Then graph the curve and this tangent line on the same axes.
2. The first way:
  - (a) Launch the *Tangents* tutor.
  - (b) Enter the function as  $x^3-2*x+1$  and the base point as 2.
  - (c) Press the **Display** button.
  - (d) The maplet will return the graph of the function and the tangent line when you press **Close**.
3. The second way:
  - (a) Enter and execute one by one the following Maple commands.
    - `f := x^3-2*x+1; or f := x->x^3-2*x+1;` Assign the function to f or define the function f(x).
    - `Df := diff(f,x); or Df := diff(f(x),x);` Compute the derivative f'(x) and assign to Df.
    - `m := eval(Df,x=2);` Find f'(2) and assign to m.
    - `L := m*(x-2)+eval(f,x=2); or L := m*(x-2)+f(2);` L is the equation of the tangent line.
    - `plot( [f, L], x=0..3 ); or plot( [f(x), L], x=0..3 );` Plot the function and the tangent line.

Now do the same for the following functions:

1.  $f(x) = x^2, x_1 = 1$
2.  $f(x) = 2^x, x_1 = 1$
3.  $f(x) = \sin(x), x_1 = \frac{\pi}{4}$  (Recall that, in Maple, you type Pi for  $\pi$ )

**C)** To practise finding the equation of the tangent line by hand, launch the *TangentLine* maplet from the course website, click on New Function, fill in blue boxes with your answers, and click on Check (or on Show if you don't know the answer). Please try a few problems to make sure that you really understand how to find the equation of tangent line by hand.

### Assignment

Exercise 41, 42, 59, and 60 of §3.6 on pages 214-215.