

Implicit Differentiation

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Overview

This lab provides experience working with functions defined implicitly. The first task is to be able to graph an implicitly-defined function. Next, the equation of the tangent line at a point on the graph of an implicitly-defined function is found and added to the graph.

Maple Essentials

- The *ImplicitDifferentiation* maplet is available from Texas A & M University at the URL:
<http://calclabs.math.tamu.edu/maple/maplets/ImplicitDifferentiation.maplet>
- The new Maple commands introduced in this project are:

Command	Description
<code>display</code>	combine one or more plots in a single plot; part of the <code>plots</code> package
<code>implicitdiff</code>	compute derivatives for implicitly-defined functions
<code>implicitplot</code>	create graph of function defined implicitly; part of the <code>plots</code> package
<code>simplify</code>	simplify an expression
<code>solve</code>	solve one or more equations for specified variables
<code>with</code>	loads the contents of a Maple package

Preparation

Review implicitly-defined functions and implicit differentiation (pp. 211–217 in Anton). Also, review the methods introduced last week for finding and plotting tangent lines (for functions defined explicitly).

Definition. Two curves are said to be *orthogonal* if their tangent lines are perpendicular at each point of intersection. Two families of curves are said to be *orthogonal trajectories* if each member of one family is orthogonal to each member of the other family.

Activities

1. Consider the problem:

Graph $y^3 + yx^2 + x^2 - 3y^2 = 0$ with a viewing window of $(-5, 5) \times (-2, 4)$. Find the slope at the point $(-1, 1)$, then add the graph of the tangent line at this point to the original graph of the curve.

This problem will be completed by entering and executing the Maple commands in Table 1 (see page 2) in a Maple worksheet.

2. Graph the curve $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ and the tangent line to this curve at the point $(3, 1)$.
3. Show that the graphs of $2x^2 + y^2 = 12$ and $y^2 = 2x$ are orthogonal by (i) finding the intersection point(s) of these curves and (ii) finding the slope of each curve at each point of intersection. Graph the two curves and their tangent lines at the point(s) of intersection.
4. Find all points where the tangent line to the graph of $x^2y - xy^2 = 2$ is horizontal or vertical.

Assignment

For Project Report 2, answer the question on page 2 of this document.

The report summarizing your analysis and answers to this question are due at the beginning of next week's lab period.

Project 2

Consider the following eight families of curves:

$$\begin{array}{cccc} (x-c)^2 + y^2 = c^2 & xy = c & x^2 + (y-c)^2 = c^2 & x^2 - xy + y^2 = c^2 \\ x-y = c(x+y)^3 & 2cy + x^2 = c^2 \ (c < 0) & x^2 - y^2 = c^2 & 2cy + x^2 = c^2 \ (c > 0) \end{array}$$

where c is a non-zero parameter.

For each family of curves, find a general formula for the slope of the curves in that family in terms of x and y but not the parameter, c . Identify the four pairs of orthogonal trajectories. For each pair of families, create a graph with representative curves from each of the two families that clearly illustrates the orthogonality. (You should have a total of four figures and each figure should contain at least eight curves.)

Additional Notes

- There are several alternate methods to obtain the plots for Activity 1. Here are the two most reasonable alternatives.
 - To use the *Plot Builder* assistant, enter two equations:

$$\begin{aligned} y^3 + yx^2 + x^2 - 3y^2 &= 0 \\ y &= -2x - 1 \end{aligned}$$
 The default plot for these functions will be a 2-D implicit plot. Specify the viewing window and request the plot. If you enter the equation of the tangent line as $-2x - 1$, Maple assumes the right-hand side is 0. In this case Maple will produce the graph of the vertical line with $-2x - 1 = 0$, i.e., $x = -\frac{1}{2}$.
 - Another approach is to use the *Plot Builder* assistant to graph the curve and find the equation of the tangent line as in Table 1. Then, highlight the equation of the tangent line, select **Copy** from the context menu, move the focus to the graph of the curve, and **Paste** the curve into the plot.
- The *ImplicitDifferentiation* maplet provides additional practice finding the slope of a curve at a point.

```
> with( plots ); # load Maple package with more plot commands
> eq := y^3 + y*x^2 + x^2 - 3*y^2 = 0; # define y, implicitly, in terms of x
> P1 := implicitplot( eq, x=-5..5, y=-2..4 ); # plot curve on window; note use of colon
> P2 := pointplot( [-1,1], color=green, # plot point on curve; note use of colon
> symbolsize=20 );
> display( [P1,P2] ); # display plot of curve and point
> DyDx := implicitdiff( eq, y, x ); # use eq to compute derivative of y with respect to x
> m := eval( DyDx, {x=-1, y=1} ); # slope of curve at (-1,1) is dy/dx |_{(x,y)=(-1,1)}
> L := m * (x+1) + 1; # tangent line is y = m(x+1) + 1
> P3 := plot( L, x=-5..5, color=blue ); # plot tangent line; note use of colon (again!)
> display( [ P1, P2, P3 ] ); # single plot with curve, point, and tangent line
```

Table 1: Sequence of Maple commands used to graph the implicitly-defined function $y^3 + yx^2 + x^2 - 3y^2 = 0$, the point $(-1, 1)$ on the curve, and the tangent line to the curve at this point.