Mathematics 788B, Introduction to Modular Forms

• Instructor: Matthew Boylan

• Course Description:

Modular forms play a central role in modern and classical number theory. Most prominently, they play a key role in Wiles’ proof of Fermat’s Last Theorem. The goal of this course is to give an elementary introduction to the theory of modular forms. There is no prerequisite.

A modular form is an analytic function \( f \) of the complex upper half-plane, \( \mathfrak{h} = \{ z : \operatorname{Im} z > 0 \} \). It possesses many self-symmetries and has an expansion in the power series variable \( q = e^{2\pi iz} \) of the form

\[
f(z) = a(0) + a(1)q + a(2)q^2 + \cdots.
\]

The coefficients \( a(n) \) often encode interesting arithmetic information. For example, modular forms naturally occur as generating functions for the following important arithmetic objects:

(1) representation numbers of positive definite quadratic forms.
(2) numbers of solutions to certain Diophantine equations (mod \( p \)) when \( p \) is prime.
(3) special values of \( L \)-functions.
(4) partition functions.
(5) invariants in algebraic number theory such as class numbers of imaginary quadratic number fields.

This course will cover modular forms on congruence subgroups of \( \text{SL}_2(\mathbb{Z}) \), Hecke operators, newforms, theta functions, \( L \)-functions, half-integer weight modular forms and Shimura’s Correspondence, and time permitting, Galois representations and modular forms modulo a prime \( \ell \).

Throughout the course, concrete examples will be given, applications will be discussed, and open problems will be mentioned.

• Text:


• Other References:

(3) Diamond, F. and Shurman, J., A First Course in Modular Forms, 1st ed., Springer-Verlag GTM 228, 2005. (Chapters 1-5)

- **Homework**: Periodic assignments (4 – 6) will be given over the semester.
- **Grading**: Based on homework. A take-home mid-term and/or final exam may be administered.