Math 784
Homework 2
Due Friday, Feb. 21.

Problems:

1. Marcus, §2, #8.

2. Marcus, §2, #22.

3. If the minimal polynomial of \( \alpha \) is \( f_\alpha(x) = x^n + ax + b \), show that for \( K = \mathbb{Q}(\alpha) \),

\[
\Delta_{K/\mathbb{Q}}(\alpha) = (-1)^{\frac{n(n-1)}{2}}(n^n b^{n-1} + a^n (1 - n)^{n-1})
\]

4. Do both of the following problems.

- Find an integral basis for the ring of integers of \( K = \mathbb{Q}(\alpha) \), where \( \alpha \) is a root of the polynomial \( x^3 - 2x + 3 \).
- Find an integral basis for the ring of integers of \( K = \mathbb{Q}(\theta) \), where \( \theta \) is a root of the polynomial \( x^3 - x + 4 \).

5. Do one of the following problems.

- Let \( K \) be a number field, let \( m \in \mathbb{Z} \), and let \( \alpha \in O_K \). Show that

\[
\Delta_{K/\mathbb{Q}}(\alpha) = \Delta_{K/\mathbb{Q}}(\alpha + m).
\]

- Suppose that \( K \) is a number field with \( r_1 \) real embeddings and \( 2r_2 \) complex embeddings so that

\[
r_1 + 2r_2 = [K : \mathbb{Q}] = n.
\]

Show that \( \Delta_K \) has sign \((-1)^{r_2} \).