Exam 3 Information  
Math 544, Sections 401 and 501

Exam 3 will be based on:

- Sections 4.3, 4.4, 4.5, 4.7, 5.2 - 5.5.
- The corresponding assigned homework problems
  (see http://www.math.sc.edu/~boylan/SCCourses/math544/544.html).
  At minimum, you need to understand how to do the homework problems.

Topic List (not necessarily comprehensive):

§4.3: **Elementary operations and determinants:** Determinant of a triangular matrix; \( \det(A) = \det(A^T) \). Effects of elementary row and column operations on the computation of a determinant (e.g., interchanging two rows changes the sign.)

§4.4: **Eigenvalues and the characteristic polynomial:** The definition and computation of the eigenvalues of a matrix \( A \in \text{Mat}_{n \times n}(\mathbb{R}) \). i.e., computation of the characteristic polynomial \( p(t) = \det(A - t I_n) \); algebraic multiplicity of an eigenvalue; eigenvalues of \( A^k \) and \( A^{-1} \); eigenvalues of \( A^T \); eigenvalues of a triangular matrix.

§4.5: **Eigenspaces and eigenvectors:** The definition and computation of the eigenvectors of a matrix \( A \in \text{Mat}_{n \times n}(\mathbb{R}) \). The definition and computation of the eigenspace and geometric multiplicity associated to a given eigenvalue of \( A \in \text{Mat}_{n \times n}(\mathbb{R}) \). The relationship between algebraic and geometric multiplicities:

\[
1 \leq \text{geometric mult.}(\lambda) \leq \text{algebraic mult.}(\lambda).
\]

Definition of a defective matrix: a matrix for which the above inequality is strict for at least one of its eigenvalues, \( \lambda \). Eigenvectors associated to distinct eigenvalues are linearly independent.

§4.7: **Similarity transformations and diagonalization:** Definition of similarity and diagonalizability. Similar matrices have the same characteristic polynomial, and hence, the same eigenvalues (but the corresponding eigenvectors are typically different!). Criterion for diagonalizability: the diagonalizability of \( A \) is equivalent to (1) \( A \) has \( n \) linearly independent eigenvectors (the maximum possible). (2) \( A \) is not defective (i.e., the geometric and algebraic multiplicities agree for all eigenvalues of \( A \)). If \( A \) is diagonalizable, determination of a matrix \( S \) which diagonalizes \( A \) and the diagonal matrix \( D \) obtained by diagonalizing with \( S \).

§5.2: **Vector spaces:** The definition of vector space (a set \( V \) and a scalar field \( F \) together with an addition operation on \( V \) and a scalar multiplication operation); in particular, the ten vector space axioms: 2 closure axioms, 4 axioms for vector addition, 4 axioms for scalar
multiplication. Examples of vector spaces: \( \text{Mat}_{m \times n}(\mathbb{R}) \), \( P_n \). Check whether a set \( V \) together with an addition and scalar multiplication is or is not a vector space.

§5.3: **Subspaces**: Definition of a subspace. Determine whether a subset \( W \) of a vector space \( V \) is a subspace: (1) is the “zero” vector in \( V \) also in \( W \)? (2) For any \( u, v \in W \), and any scalar \( c \), is \( cu + v \in W \)? If \( W \) is a subspace, definition of what it means for a subset \( Q \subset W \) to span \( W \). Since \( W \) is a subspace, a subset \( Q \) spans \( W \) if and only if \( \text{Sp}(Q) = W \), where \( \text{Sp}(Q) \) is the set of all linear combinations of vectors from \( Q \). Given a subspace \( W \), find a subset \( Q \) which spans it.

§5.4: **Linear independence, bases, and coordinates**: Definition of linear dependence / independence, basis of a vector space \( V \), ordered basis for a vector space \( V \), coordinates of a vector \( v \in V \) relative to an ordered basis \( B \) for \( V \).

Given a subset of vectors \( Q \subset W \), determination of whether \( Q \) is linearly independent. Given a subspace \( W \) of a vector space \( V \), determination of a basis for \( W \). Given a basis \( B \) for a vector space \( V \) and a vector \( v \in V \), determination of \( (v)_B \), the coordinates of \( v \) relative to the basis \( B \).

§5.5: **Dimension**: Definition and computation of the dimension of a subspace \( W \) of a vector space \( V \).