Exam 2 Information  
Math 544, Sections 401 and 501

Exam 2 will be based on:

- Sections 3.3 - 3.7, 4.1, 4.2.
- The corresponding assigned homework problems  
  (see http://www.math.sc.edu/~boylan/SCCourses/math544/544.html).
  
  At minimum, you need to understand how to do the homework problems.
- Lecture notes: 2/6 - 3/3.

Topic List (not necessarily comprehensive):

§3.3: **Examples of subspaces:** Let \( A \in \text{Mat}_{m \times n}(\mathbb{R}) \). Important subspaces associated to \( A \) are: **Null space** of \( A \) (subspace of \( \mathbb{R}^m \)), **range** of \( A \) (subspace of \( \mathbb{R}^n \)) (also called the **column space** of \( A \)), **row space** of \( A \) (subspace of \( \mathbb{R}^n \)). For a matrix \( A \), give an algebraic description of the vectors in each of these subspaces. If \( A \) and \( B \) are row equivalent, they have the same row space.

§3.4: **Bases for subspaces:** Definition of a basis for a subspace \( W \subseteq \mathbb{R}^n \): A **linearly independent set** which spans \( W \). Example: the standard basis, \( \{e_1, \ldots, e_n\} \). Computation of bases for: Null(\( A \)), Range(\( A \)), Row(\( A \)):

- **Row(\( A \))**:  
  Put \( A \) in row echelon form, \( B \). The nonzero rows form a basis for Row(\( A \)).

- **Range(\( A \))**:  
  If you want a basis consisting of columns of \( A \), you put \( A \) in reduced row echelon form, \( B \). The columns of \( B \) with the leading 1’s correspond to the columns of \( A \) which go into the basis.
  
  If you don’t care about where you pick your basis vectors from, you can put \( A^T \) in row echelon form. The nonzero rows for a basis form Row(\( A^T \))=Range(\( A \)).

- **Null(\( A \))**:  
  Solve \( Ax = \theta \) to determine a spanning set for Null(\( A \)). Then check whether the spanning set is linearly independent.

§3.5: **Dimension:** Bases for subspaces are not unique, but the number of vectors in any (all) bases for a subspace \( W \) is the same; this number is the dimension of \( W \). Theorem 9, page 207. **Rank** and **nullity** of a matrix \( A \in \text{Mat}_{m \times n}(\mathbb{R}) \), **rank**(\( A \)) + **nullity**(\( A \)) = \( n \). **Rank**(\( A \)) = \( \text{dim Range}(A) = \text{dim Row}(A) \). \( A \in \text{Mat}_{n \times n}(\mathbb{R}) \) is non-singular if and only if **rank**(\( A \)) = \( n \) (and hence **nullity**(\( A \)) = 0).
\section*{3.6: Orthogonal bases for subspaces:} Orthogonal and orthonormal sets of vectors. “Orthonormalizing” an orthogonal set. Finding coordinates for a vector in terms of an orthogonal basis. Existence of an orthogonal basis.

\section*{3.7: Linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^m$:} Definition of a linear transformation, how to check whether a given map is a linear transformation or not. $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a linear transformation if and only if for every $x \in \mathbb{R}^n$, $T(x) = Ax$, where

$$A = [T(e_1) \cdots T(e_n)]$$

(i.e., it suffices to know what $T$ does to the standard basis for $\mathbb{R}^n$). \textbf{Null space} and \textbf{Range} of $T$.

\section*{4.1: The eigenvalue problem for $2 \times 2$ matrices:} Definition and computation of eigenvalues and eigenvectors of a matrix $A \in \text{Mat}_{2 \times 2}(\mathbb{R})$.

\section*{4.2: Determinants and the eigenvalue problem:} Determinants of matrices $A \in \text{Mat}_{n \times n}(\mathbb{R})$. Computation of determinants using \textbf{minors} and \textbf{cofactors}. Properties of determinants, for example:

- $\det(AB) = \det(A)\det(B)$.
- $A \in \text{Mat}_{n \times n}(\mathbb{R})$ is singular $\iff \det(A) = 0$. 
