Math 546/70I, Homework 5 Problems.

You are expected to do every assigned homework problem. You should review the relevant sections of the text and review the relevant class notes. Your solutions must be clearly written and neatly prepared. In particular, your pages must be stapled together, and must not have rough edges.

Due 10/5: Beachy and Blair §3.3: 1, 3 - 8, and do either 11 or 14.

**Challenge problems:** Attempt these only after you have tried the problems above.
You can try some, all, or none of the challenge problems.

- Do **both** #11 and #14.

- 15, 16, 17. These problems show that there only two distinct structures a group of order 6 could have, and examples of each type are given by $\mathbb{Z}_6$ and $S_3$. Note that we showed in class that there are only two distinct structures a group of order 4 could have, and examples of each type are given by $\mathbb{Z}_4$ and $\mathbb{Z}_8^\times$. The order 6 case is a bit harder.

- Let $p$ be prime. Show that the group $GL_2(\mathbb{Z}_p)$ has order

  $$|GL_2(\mathbb{Z}_p)| = (p^2 - 1)(p^2 - p).$$