**Math 241, Quiz 3. 2/4/13.**

- Read problems carefully. Show all work. No notes, calculator, or text.
- There are 15 points total.

1. §12.4, #38 (7 points): Use the scalar triple product to determine whether the points $A = (1, 3, 2)$, $B = (3, -1, 6)$, $C = (5, 2, 0)$, $D = (3, 6, -4)$ lie in the same plane.

   **Solution:** We have
   
   \[ \vec{AB} = (2, -4, 4), \quad \vec{AC} = (4, -1, -2), \quad \vec{AD} = (2, 3, -6). \]
   
   The scalar triple product is
   
   \[
   \vec{AB} \cdot (\vec{AC} \times \vec{AD}) = \begin{vmatrix}
   2 & -4 & 4 \\
   4 & -1 & -2 \\
   2 & 3 & -6
   \end{vmatrix} = 2(6 - (-6)) - (-4)(-24 - (-4)) + 4(12 - (-2))
   \]
   
   \[ = 24 - 80 + 56 = 0. \]
   
   Since the triple product measures the volume of the parallelepiped determined by the vectors, zero volume implies that the vectors are coplanar.

2. §12.5, #22 (8 points): Determine whether the lines
   
   \[ L_1 : \frac{x - 1}{2} = \frac{y - 3}{2} = \frac{z - 2}{-1}; \quad L_2 : \frac{x - 2}{1} = \frac{y - 6}{-1} = \frac{z + 2}{3} \]
   
   are parallel, skew, or intersecting. If they intersect, find the point of intersection.

   **Solution:** We rewrite the equations in parametric form. For all $t_1, t_2 \in \mathbb{R}$, we have
   
   \[ L_1 : x = 1 + 2t_1, \quad y = 3 + 2t_1, \quad z = 2 - t_1; \quad L_2 : x = 2 + t_2, \quad y = 6 - t_2, \quad z = -2 + 3t_2. \]
   
   The direction vectors are $\vec{v}_1 = (2, 2, -1)$, $\vec{v}_2 = (1, -1, 3)$. Since no $k \neq 0$ in $\mathbb{R}$ has $k\vec{v}_1 = \vec{v}_2$, we see that the lines are not parallel. To conclude we check whether they are skew or intersecting. We solve for $t_1, t_2$ in
   
   \[ 1 + 2t_1 = 2 + t_2, \quad 3 + 2t_1 = 6 - t_2, \quad 2 - t_1 = -2 + 3t_2. \]
   
   Adding the first and second equations gives $4 + 4t_1 = 8$, which yields $t_1 = 1$. We substitute $t_1 = 1$ in the first equation to get $t_2 = 1$. These values correspond to a point of intersection since they satisfy the third equation, $2 - t_1 = -2 + 3t_2$. Hence, the lines intersect. One obtains the point of intersection by substituting $t_1 = 1$ in the equations for $L_1$: $(1 + 2t_1, 3 + 2t_1, 2 - t_1) = (3, 5, 1)$, or by substituting $t_2 = 1$ in the equations for $L_2$. 