Math 241, Quiz 6. 2/20/12. Name: ________________

- Read problems carefully. Show all work. No notes, calculator, or text.
- There are 15 points total.

1. §14.3, #27 (5 points): Let \( u = te^{w/t} \). Find \( \frac{\partial u}{\partial t} \).

   \[
   \frac{\partial u}{\partial t} = t \frac{\partial}{\partial t}(e^{w/t}) + e^{w/t} \frac{\partial}{\partial t}(t) = te^{w/t} \cdot (\frac{w}{t^2}) + e^{w/t} = e^{w/t} - \frac{w}{t} \cdot e^{w/t} = e^{w/t} \left(1 - \frac{w}{t}\right).
   \]

2. §14.3, #51 (5 points): Find all second partial derivatives of

   \[
   f(x, y) = x^3 y^5 + 2x^4 y.
   \]

   \[
   \text{Solution: } f_x = 3x^2 y^5 + 8x^3 y; \quad f_y = 5x^3 y^4 + 2x^4; \quad f_{xx} = 6x y^5 + 24x^2 y; \quad f_{xy} = 15x^2 y^4 + 8x^3; \quad f_{yy} = 20x^3 y^3.
   \]

3. §14.4, #3 (5 points): Find an equation of the tangent plane to the surface

   \[
   z = \sqrt{xy}
   \]

   at the point \((1, 1, 1)\). Write you answer in the form \(ax + by + cz + d = 0\).

   \[
   \text{Solution: } f_x = (1/2)(xy)^{-1/2}; \quad f_y = (1/2)\sqrt{y/x} \quad \Rightarrow \quad f_x(1, 1) = 1/2; \quad f_y(1, 1) = 1/2. \quad \text{Equation of the tangent plane:}
   \]

   \[
   z - 1 = (1/2)(x - 1) + (1/2)(y - 1) \quad \iff \quad 2z - 2 = (x - 1) + (y - 1) \quad \iff \quad x + y - 2z = 0.
   \]