
Name: ____________________

- Read problems carefully. Show all work.
- No notes.
- The exam is approximately 15 percent of the total grade.
- There are 100 points total. Partial credit may be given.
1. (10 points) Find an equation of the tangent plane to

\[ xyz + 2x + 3y + 3z + 2 = 0 \]

at \((1, 2, -2)\).
2. (20 points) Locate all \textbf{relative} maxima, minima, and saddle points for

\[ f(x, y) = x^2 - xy + y^2 + 2x + 2y - 4. \]
3. (20 points) Use Lagrange multipliers to find the point(s) \((x, y, z)\) with \(x, y, z \geq 0\) which maximize \(f(x, y, z) = xy^2z^2\) subject to the constraint \(g(x, y, z) = x + y + z - 5 = 0\).
4. (10 points) Let $R$ be the region in the $xy$-plane enclosed by

$$y = x^2, \quad y = \frac{1}{x}, \quad \text{and} \quad y = 4.$$ 

**Set up, but do not evaluate**, an iterated integral (or sum of iterated integrals) which gives the **area** enclosed by the region $R$.

You may view the region $R$ as a type I region (integrate with respect to $y$ first) or type II region (integrate with respect to $x$ first).
5. **(15 points)** Consider the iterated integral

\[ \int_{0}^{4} \int_{\sqrt{y}}^{2} \sqrt{x^3 + 1} \, dx \, dy. \]

(a) **(8 points)** Express the iterated integral as an equivalent iterated integral with the order of integration reversed. You may want to first sketch the region \( R \) over which the integral is taken.

(b) **(7 points)** Evaluate the iterated integral that you obtained in part (a).
6. **(10 points) Set up, but do not evaluate** an iterated integral in rectangular coordinates which gives the volume of the solid enclosed by

- \( z = 0 \) (the \( xy \)-plane),
- the plane \( y = z \),
- and the cylinder \( x^2 + y^2 = 4 \).

It is a good idea (but not essential) to first sketch the solid.
7. (15 points) Consider the iterated integral

\[ \int_0^3 \int_0^{\sqrt{9-x^2}} e^{x^2+y^2} \, dy \, dx. \]

(a) (8 points) Express the integral as an equivalent integral in polar coordinates. You may want to first sketch the region over which the integral is taken.

(b) (7 points) Evaluate the iterated integral that you obtained in part (a).